Ultrasonics 54 (2014) 106–120

Contents lists available at SciVerse ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Reliability assessment of different plate theories for elastic wave propagation analysis in functionally graded plates

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ARTICLE INFO

Article history:
Received 4 July 2012
Received in revised form 8 January 2013
Accepted 29 April 2013
Available online 9 May 2013

Keywords:
Elastic wave propagation
FGM plate
Classical plate theory (CPT)
First-order shear deformation theory (FSDT)
Third-order shear deformation theory (TSDT)

ABSTRACT

The importance of elastic wave propagation problem in plates arises from the application of ultrasonic elastic waves in non-destructive evaluation of plate-like structures. However, precise study and analysis of acoustic guided waves especially in non-homogeneous waveguides such as functionally graded plates are so complicated that exact elastodynamic methods are rarely employed in practical applications. Thus, the simple approximate plate theories have attracted much interest for the calculation of wave fields in FGM plates. Therefore, in the current research, the classical plate theory (CPT), first-order shear deformation theory (FSDT) and third-order shear deformation theory (TSDT) are used to obtain the transient responses of flexural waves in FGM plates subjected to transverse impulsive loadings. Moreover, comparing the results with those based on a well recognized hybrid numerical method (HNM), we examine the accuracy of the plate theories for several plates of various thicknesses under excitations of different frequencies. The material properties of the plate are assumed to vary across the plate thickness according to a simple power-law distribution in terms of volume fractions of constituents. In all analyses, spatial Fourier transform together with modal analysis are applied to compute displacement responses of the plates. A comparison of the results demonstrates the reliability ranges of the approximate plate theories for elastic wave propagation analysis in FGM plates. Furthermore, based on various examples, it is shown that whenever the plate theories are used within the appropriate ranges of plate thickness and frequency content, solution process in wave number-time domain based on modal analysis approach is not only sufficient but also efficient for finding the transient waveforms in FGM plates.

1. Introduction

For decades, laminated composite plates have been used in various applications, especially in aerospace and automotive industries all over the world. However, the predominant drawback of such materials is their layered structure that makes them prone to the phenomenon identified as delamination, particularly under some extreme loading conditions such as impact loading. To overcome this problem, functionally graded materials were proposed as a remedy in the middle of 1980s by Japanese researchers [1,2]. FGMs are inhomogeneous composites with smooth and continuous grading of volume fractions of constituents. In general, the ingredients are ceramic and metal for which the grading, from one side to the other, takes place prevalently in the thickness direction of a plate. Correspondingly, by preventing an abrupt change in the mechanical properties, the problems of thermal and residual stresses as well as stress concentration are removed that can be the main reasons for delamination. Ceramic as a thermal barrier material with low density and considerable toughness can play a protective role for metal against corrosion and oxidation. In addition, metal provides strength, ductility and thermal conductivity for a FGM plate. The above-mentioned remarkable properties have enabled functionally graded material plates to sustain severe thermo-mechanical loadings which are common in some important industries like aerospace, nuclear or defense. Consequently, FGM plates have found widespread applications in manufacturing nuclear fusions, gas turbines, thermal barrier structures, chemical plants, etc.

On the other hand, non-destructive evaluation methods are widely used for damage detection in engineering structures. The NDE procedures based on ultrasonic elastic waves have attracted much attention for structural health monitoring of plates in recent years. Performing these processes requires a deep understanding of transient elastic wave motion in plates. The research works on the elastodynamics of homogeneous plates are numerous; however, the literature on elastic wave propagation in non-homogeneous FGM plates is scarce.

Liu et al. [3] developed a hybrid numerical method for functionally gradient material plates to describe some properties of Lamb

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http://dx.doi.org/10.1016/j.ultras.2013.04.022

waves in non-homogeneous plates. Chiu and Erdogan [4] considered one-dimensional problems in elastodynamics for a FGM plate in which the boundary conditions were assumed to be either free/free or fixed/free. Then, using the Laplace transform technique they solved the problem under zero initial conditions with a rectangular pressure pulse as the external load. Liu et al. [5] suggested a method to analyze the response of a FGM plate excited by plane pressure wavelets. Dividing the FGM plate into linearly inhomogeneous elements, they obtained dynamic response in the frequency domain and then using Fourier transform techniques, the response of the FGM plate was presented. Lefebvre et al. [6] expanded the Legendre polynomials and without dividing the plate into layers studied the propagation of guided waves in continuous functionally graded plates. Han et al. [7] extended the hybrid numerical method for analyzing transient waves in FGM plates excited by impact loads. Han and Liu [8] employed quadratic layer elements to study SH waves in FGM plates. They also presented a simple and novel method to evaluate the modified Bessel functions with complex valued orders and arguments which are involved in the solution procedure of these problems. Berezovski et al. [9] studied 2D elastic stress wave propagation in two models of layered and graded FGM plates. They compared the time evolutions of the field quantities as a result of wave propagation and interaction with interfaces and gradients in these two models and clarified the significance of grading schemes on impact applications. Wave propagation in inhomogeneous layered media was studied by Chakraborty and Gopalakrishnan [10], who used the method of partial wave technique in conjunction with linear algebraic methodology. Chen et al. [11] studied the dispersion behavior of waves in a functionally graded elastic plate. They showed that the combination of reverberation matrix and state-space methods is an efficient solution for calculating dispersion curves over a wide range of wave numbers. Based on homotopy analysis method (HAM), Gao et al. [12] studied surface acoustic waves in an elastic FGM plate, having coupled differential equations with variable coefficients. They also compared the effectiveness of the analytical methods, including a layered model, the Frobenius method and the HAM. Yu et al. [13] investigated the propagation of thermoelastic waves in continuous orthotropic FGM plates in the context of the Green–Naghdi generalized thermoelastic theory. In their research, the coupled wave equation and heat conduction equation were solved by the Legendre orthogonal polynomial series expansion approach. Using a Legendre orthogonal polynomial series approach, Yu et al. [14] presented a dynamic solution for the propagation of guided waves in continuous viscoelastic FGM plates in the context of Kelvin–Voigt viscoelastic theory. Cao et al. [15] employed the power series technique to analyze the propagation behavior of Lamb waves in a thermal stress relaxation type functionally graded material plate. By means of integral transforms and based on a higher-order shear deformation theory of plates, Sun and Luo [16] studied the wave propagation and transient response of an infinite functionally graded plate under a point impact load. Furthermore, they carried out a similar investigation for FGM plates in thermal environments [17].

The present work investigates the efficiency and accuracy of plate theories for estimating flexural waveforms of functionally graded material plates in high frequency analyses. Earlier, Lih and Mal [18] presented an outlook for the accuracy of the first-order shear deformation plate theory in analyzing of flexural waves in composite laminates. They showed that in addition to static and vibration analysis of laminates with finite dimensions, the laminate theory could also be used for analyzing wave propagation problems in the laminates of large lateral dimensions. They employed a multiple transform technique coupled with a numerical inversion scheme for resulting double integral expressions to calculate the displacements produced in a composite laminate subjected to dynamic loads. This method was proposed in their previous research [19] in which the quantitative features of the elastodynamic field in composite laminates produced by surface loads, were studied by means of the multiple transform technique. Due to dealing with equations of motion transformed into the frequency-wave number domain, they had to tackle with the difficulties of inverse transformation process back into the time–space domain. In their solution method, the dissipation property of material was an asset to keep the solutions away from resulting singularities in integrations which were inevitable in the absence of material dissipation. After the suggestion of hybrid numerical method for elastodynamic analysis of FGM plates, the advantages of wave propagation analysis in time domain were again highlighted; nevertheless, the use of this approach based on plate theories, particularly for FGM plates, has not been reported in the previous researches so far.

This article, for the first time, by developing extensive relations, employs the above mentioned approach for flexural wave propagation analysis in FGM plates based on plate theories. It is shown that whenever the plate theories are used, solution process in the wave number-time domain can be not only sufficient but also efficient for finding the transient waveforms in FGM plates. This study demonstrates the possibility of avoiding temporal transformations that are major challenges in the solution procedures of the previously related studies. Besides, by using this method, detailed and comprehensive comparisons among the classical, first-order and third-order shear deformation theories of plates are presented for evaluating flexural waveforms of FGM plates. In addition, the results based on hybrid numerical method are used as benchmarks for comparing with those of plate theories. To the best knowledge of authors, these comparisons have not been performed yet. The comparisons among these theories could be of great importance, because there could always be serious concerns about the consequences resulting from these simplifying assumptions on the numerical results, which are the basis of plate theories. Whenever the material inhomogeneity is involved, this issue might be intensified. In other words, the problem of acceptable degree of simplifications in the elastodynamic analyses could be worthwhile for extensive research. This article could be assumed as a first attempt to alleviate this concern. Several examples are presented for some types of impulsive loadings applied to FGM plates of different thicknesses and volume fractions to illuminate the sensitivity of plate theories to the plate thickness as well as frequency contents of the dynamic excitations.

2. Mathematical model for grading of FGM plate

An infinite ceramic–metal functionally graded material plate of thickness h is considered. The xy-plane of coordinate system is set at the mid-plane of the plate. The top surface of the plate ($z = +h/2$) is ceramic-rich, whereas the bottom one ($z = -h/2$) is metal-rich. Within the top and bottom surfaces of the FGM plate, there is a combination of ceramic and metal with gradual variation of the constituents in the thickness direction (See Fig. 1). The mechanical properties p of the FGM plate such as Young's modulus E and mass density ρ can be expressed by a simple power-law distribution in terms of the volume fractions of the constituents [20]:

$$p_z = p_m + (p_c - p_m)\left(\frac{2z + h}{2h}\right)^N \tag{1}$$

where subscripts m and c refer to metal and ceramic, respectively. Further, z is the thickness coordinate and N denotes the volume fraction index of the FGM plate that takes values greater than or equal to zero. $N = 0$ and infinite N represent isotropic ceramic and