Ultrasonics 54 (2014) 351–357

Contents lists available at SciVerse ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Mechanism of the quasi-zero axial acoustic radiation force experienced by elastic and viscoelastic spheres in the field of a quasi-Gaussian beam and particle tweezing

F.G. Mitri [a,*], Z.E.A. Fellah [b]

[a] Los Alamos National Laboratory, MPA-11, Sensors & Electrochemical Devices, Acoustics & Sensors Technology Team, MS D429, Los Alamos, NM 87545, USA
[b] Laboratoire de Mécanique et d'Acoustique, CNRS-UPR 7051, 31 chemin Joseph Aiguier, Marseille 13402, France

ARTICLE INFO

Article history:
Received 23 January 2013
Received in revised form 3 April 2013
Accepted 3 April 2013
Available online 22 April 2013

Keywords:
Radiation force
Quasi-Gaussian beam
Acoustic scattering
Elastic and viscoelastic sphere

ABSTRACT

The present analysis investigates the (axial) acoustic radiation force induced by a quasi-Gaussian beam centered on an elastic and a viscoelastic (polymer-type) sphere in a nonviscous fluid. The quasi-Gaussian beam is an exact solution of the source free Helmholtz wave equation and is characterized by an arbitrary waist w_0 and a diffraction convergence length known as the Rayleigh range z_R. Examples are found where the radiation force unexpectedly approaches closely to zero at some of the elastic sphere's resonance frequencies for $kw_0 \leqslant 1$ (where this range is of particular interest in describing strongly focused or divergent beams), which may produce particle immobilization along the axial direction. Moreover, the (quasi)vanishing behavior of the radiation force is found to be correlated with conditions giving extinction of the backscattering by the quasi-Gaussian beam. Furthermore, the mechanism for the quasi-zero force is studied theoretically by analyzing the contributions of the kinetic, potential and momentum flux energy densities and their density functions. It is found that all the components vanish simultaneously at the selected ka values for the nulls. However, for a viscoelastic sphere, acoustic absorption degrades the quasi-zero radiation force.

1. Introduction

Quasi-Gaussian beams have been recently originated in the wave diffraction theory as an exact solution of the Helmholtz equation. The properties of such beams have been analyzed from the standpoint of the classical wave propagation theory based on the complex source point method [1–8] to obtain the expression of the pressure for the incident quasi-Gaussian beam, and expand it using a partial-wave series [9,10]. A quasi-Gaussian beam (Fig. 1) is characterized by an arbitrary waist w_0 and a diffraction convergence length known as the Rayleigh range z_R. Moreover, the beam has the form of a superposition of sources and sinks with complex coordinates [9].

In a recent investigation [11], the scattering (which is an important phenomenon in many applications, for example nondestructive imaging applications [12,13], medical imaging etc.), instantaneous and mean radiation forces experienced by a rigid and immovable (fixed) sphere centered on the axis of the beam have been investigated theoretically. Situations have been observed where significant differences have occurred between the quasi-Gaussian beam and the plane wave results for $kw_0 < 25$, (where k denotes the wavenumber of the incident beam), however, the plane wave results have been recovered when $kw_0 > 25$ and increases toward $\rightarrow \infty$.

The purpose here is to illustrate situations where the radiation force function (which the radiation force per unit energy density and unit cross-section) tends to zero at some of the resonance frequencies of an *elastic* sphere and specific values of kw_0. The formalism for the scattering derived previously [11] is used here to evaluate the acoustic radiation force of a quasi-Gaussian beam on an elastic sphere in a nonviscous fluid, and correlate the backscattering and radiation force function plots. Moreover, the mechanism for the quasi-zero force is studied theoretically by analyzing the contributions of the kinetic, potential and momentum flux energy densities and their density functions. Additional examples are provided for a (polymer-type) viscoelastic sphere. The extension of the previous work [11] to account for the sphere's elasticity may be helpful for the identification of some conditions where ultrasonic quasi-Gaussian beams may be used to immobilize a sphere (or a spherical shell, a layered sphere [14–16], or a layered spherical shell [17]) in a fluid with negligible viscosity. It is important to identify such conditions using *a priori* information obtained from theoretical predictions since it may be experimentally easier to verify the existence of zero acoustic radiation forces in quasi-Gaussian beams using solid objects.

* Corresponding author.
E-mail addresses: mitri@ieee.org, mitri@lanl.gov (F.G. Mitri).

0041-624X/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ultras.2013.04.010

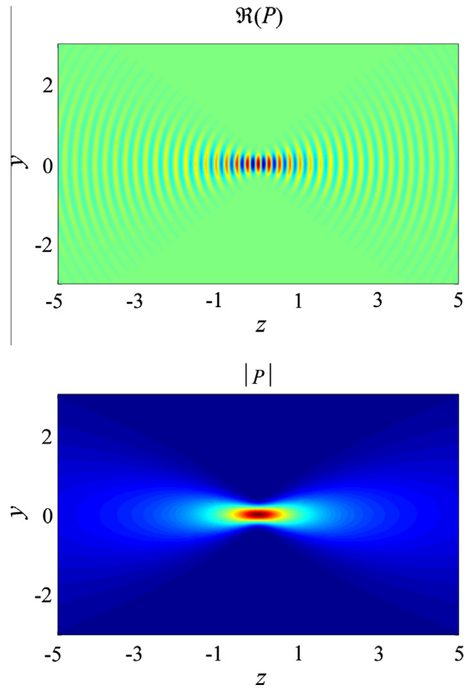

Fig. 1. Instantaneous sound pressure (top panel) for a quasi-Gaussian beam at $kw_0 = 5$. The bottom panel represents the magnitude of the pressure for $k = 25 \times 10^3\ \mathrm{m}^{-1}$. The units along the axes are in mm. (see also the Supplementary Animation).

2. Radiation force, its components and density functions

The mean (time-averaged) radiation force of a quasi-Gaussian beam of continuous waves is expressed as [18,19],

$$\langle \mathbf{F}_{rad} \rangle = \iint_{S_0} \langle \mathscr{L} \rangle \mathbf{n} dS - \iint_{S_0} \langle \rho \mathbf{v}^{(1)} (\mathbf{v}^{(1)} \cdot \mathbf{n}) \rangle dS, \tag{1}$$

where

$$\mathscr{L} = \frac{\rho_0}{2} |\mathbf{v}^{(1)}|^2 - \frac{1}{2\rho_0 c_0^2} p^{(1)^2} = \mathscr{K} - \mathscr{U}, \tag{2}$$

is the Lagrangean energy density, the superscript$^{(1)}$ denotes first-order quantities, $\mathbf{v}^{(1)} = \nabla\varphi$, $p^{(1)} = -\rho_0 \frac{\partial \varphi^{(1)}}{\partial t}$, and $\varphi^{(1)} = \mathrm{Re}[\Phi^{(1)}]$, where $\Phi^{(1)}$ is the total (incident + scattered) linear velocity potential that is related to the total pressure in the surrounding fluid.

This equation can be rewritten in terms of the following factors [20],

$$\langle \mathbf{F}_{rad} \rangle = \iint_{S_0} \langle \mathscr{K} \rangle \mathbf{n} dS - \iint_{S_0} \langle \mathscr{U} \rangle \mathbf{n} dS - \iint_{S_0} \langle \mathscr{R} \rangle dS, \tag{3}$$

where $\mathscr{R} = \rho_0 v^{(1)} \mathbf{v}_n^{(1)}$, is defined as the transmission momentum flux energy density through the sphere, and $v_n^{(1)}$ is the normal component of the velocity. The three components of the radiation force on an elastic sphere can be represented in terms of the total velocity potential $\Phi^{(1)}$ given by the partial-wave series as,

$$\varphi^{(1)} = \mathrm{Re}[\Phi^{(1)}] = \sum_{n=0}^{\infty} \Phi_0 (2n+1) R_n P_n(\cos\theta), \tag{4}$$

where Φ_0 is the (real) amplitude. The coefficient R_n is given by [11],

$$R_n = \mathrm{Re}[i^n (U_n(kr) + iV_n(kr)) g_n(kz_R) e^{-i\omega t}], \tag{5}$$

and,

$$\begin{aligned} U_n &= (1+\alpha_n) j_n(kr) - \beta_n y_n(kr), \\ V_n &= \beta_n j_n(kr) + \alpha_n y_n(kr), \end{aligned} \tag{6}$$

where $y_n(.)$ are the spherical Neumann functions (or the spherical Bessel functions of the second kind), $\alpha_n = \mathrm{Re}[S_n]$, $\beta_n = \mathrm{Im}[S_n]$, and S_n are the scattering coefficients determined by applying appropriate boundary conditions at the interface fluid–structure, with the assumption that the surrounding fluid is nonviscous. These functions depend on the sphere's elastic parameters such as the longitudinal c_L, the shear or transverse c_T sound speed and the mass densities of both the fluid ρ_0 and the sphere ρ_s. It should be emphasized that those coefficients are found equivalent to those obtained from the study of acoustic scattering by plane waves (see Appendix in [21]).

The three components of the radiation force are now expressed as [20],

$$\begin{aligned} \iint_{S_0} \langle \mathscr{K} \rangle \mathbf{n} dS &= \pi\rho_0 a^2 \left(\frac{1}{a^2} \int_0^{\pi} \left\langle \left(\frac{\partial \Phi^{(1)}}{\partial \theta} \right)^2_{r=a} \right\rangle \sin\theta\cos\theta d\theta \right. \\ &\quad \left. + \int_0^{\pi} \left\langle \left(\frac{\partial \Phi^{(1)}}{\partial r} \right)^2_{r=a} \right\rangle \sin\theta\cos\theta d\theta \right) \\ &= 2\pi\rho_0 |\Phi_0|^2 \sum_{n=0}^{\infty} \left\{ g_n(kz_R) g_{n+1}(kz_R)(n+1) \right. \\ &\quad \times [n(n+2)(V_n U_{n+1} - U_n V_{n+1}) \\ &\quad \left. + (ka)^2 (V_n' U_{n+1}' - U_n' V_{n+1}')] \right\}_{r=a}, \end{aligned} \tag{7}$$

$$\begin{aligned} \iint_{S_0} \langle \mathscr{U} \rangle \mathbf{n} dS &= \frac{\pi\rho_0 a^2}{c_0^2} \int_0^{\pi} \left\langle \left(\frac{\partial \Phi^{(1)}}{\partial t} \right)^2_{r=a} \right\rangle \sin\theta\cos\theta d\theta \\ &= 2\pi\rho_0 |\Phi_0|^2 (ka)^2 \sum_{n=0}^{\infty} \left\{ g_n(kz_R) g_{n+1}(kz_R)(n+1) \right. \\ &\quad \left. \times (V_n U_{n+1} - U_n V_{n+1}) \right\}_{r=a}, \end{aligned} \tag{8}$$

$$\begin{aligned} \iint_{S_0} \langle \mathscr{R} \rangle dS &= -2\pi\rho_0 a^2 \left(\frac{1}{a} \int_0^{\pi} \left\langle \left(\frac{\partial \Phi^{(1)}}{\partial r} \right)_{r=a} \left(\frac{\partial \Phi^{(1)}}{\partial \theta} \right)_{r=a} \right\rangle \sin^2\theta d\theta \right. \\ &\quad \left. + \int_0^{\pi} \left\langle \left(\frac{\partial \Phi^{(1)}}{\partial r} \right)^2_{r=a} \right\rangle \sin\theta\cos\theta d\theta \right) \\ &= -2\pi\rho_0 ka |\Phi_0|^2 \sum_{n=0}^{\infty} \left\{ g_n(kz_R) g_{n+1}(kz_R)(n+1) \right. \\ &\quad \times \left[n(V_n U_{n+1}' - U_n V_{n+1}') - (n+2)(V_n' U_{n+1} - U_n' V_{n+1}) \right. \\ &\quad \left. \left. -2(ka)(V_n' U_{n+1}' - U_n' V_{n+1}') \right] \right\}_{r=a}. \end{aligned} \tag{9}$$

Denoting by $E = \rho k^2 |\Phi_0|^2/2$ the characteristic energy density, the axial time-averaged radiation force of a quasi-Gaussian beam is expressed by [11],

$$\langle F_{z,rad} \rangle = Y_{qG} S_c E, \tag{10}$$

where $S_c = \pi a^2$ is the cross-sectional area, and Y_{qG} is the radiation force function, which is the radiation force per unit energy density and unit cross-sectional surface given by [11],

$$\begin{aligned} Y_{qG} = -\frac{4}{(ka)^2} \sum_{n=0}^{\infty} \{ & g_n(kz_R) g_{n+1}(kz_R)(n+1)[\alpha_n + \alpha_{n+1} \\ & + 2(\alpha_n \alpha_{n+1} + \beta_n \beta_{n+1})] \}. \end{aligned} \tag{11}$$