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Transmission analysis of ultrasonic Lamb mode conversion in a plate with partial-thickness notch

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ABSTRACT

Mode conversions of Lamb waves can occur upon encountering damage or defect such as a notch, leading to newly-converted modes apart from wave reflection and transmission. In this paper, the transmission of the fundamental Lamb modes symmetrical S0 and anti-symmetrical A0 with anti-symmetrical notches were investigated in steel plates within the relatively short propagation distance. The group velocity and modal energy of the converted modes were analyzed using simulations and experiments. Two-dimensional finite difference time domain (2D-FDTD) method was employed to calculate the scattering field and extract numerical trends for simulation study and experimental confirmation. Both simulations and experiments revealed that the apparent group velocities of the converted modes in the transmitted signals subject to the notch positions. To describe the mode conversion degree and evaluate the notch severity, wave packets of the originally-transmitted modes and newly-converted modes were separated and corresponding mode energy percentages were analyzed at different notch severities. Frequency-sweeping measurements illustrated that the modal energy percentages varied monotonically over the notch-depth increase with a statistically consistency ($R = 1.00$, $P < 0.0004$).

1. Introduction

Lamb waves can propagate through the whole thickness of the thin plate or shell structures and provide rich information of the structure health, which has been widely applied into the fast and nondestructive inspection of the large-scale structures [1–5] and also some biomaterial evaluation [6–9]. However, due to the mode conversion occurred upon the defects, Lamb waves are always recorded as combinations of multiple spreading and overlapping modes, which highly complicates the experimental signal interpretation and damage evaluation.

As one of the classic topics of the guided waves based nondestructive evaluation, the scattering or mode conversion of the Lamb waves under different conditions has been reported by wide range of studies, for instance, holes on the submerged plates [10], detection and imaging of corrosions [11], scattered field in the delamination area [12], diffraction and attenuation of guided modes in the viscoelastic material plates [13], quantitative evaluation of crack orientation [14], and mode conversion in the plate with irregular defects [15]. Specifically speaking, a number of studies of the mode interaction with notches have also been carried out. Alleyne and Cawley have analyzed the modal sensitivity of the notched plate evaluation by experimental and numerical investigations [16]. The interactions of the fundamental Lamb waves with symmetrical and asymmetrical discontinuities have been documented by Benmeddour et al. [17,18]. Recently, Castaings et al. [19] reported that transmission and reflection coefficients measured in the scattering field can be inversely used to size the cracked zones. However, in some engineering practice, the distance between the receiver and transmitter is not sufficiently long, and it is relatively difficult to separate the mixed modes. The sensor-pair, which is attached on the two opposite sides of the plate at the same location, has been employed to selectively excite and extract the pure anti-symmetrical and symmetrical modes [17,20–22]. This technique also can be used to process the mixed multimode measured in short propagation distance. However, even with the pure modes, it is clearly of great value to distinguish the wave-packet originated from mode conversion on the defects and from the direct transmission through the normal structure [23]. Therefore, further analysis of originally-transmitted and newly-converted Lamb modes is definitely required for better understanding of the mechanism of mode conversion.

The aim of this study is to clarify the originally-transmitted modes and newly-converted modes in transmission and further discuss the velocity variation and energy conversion of the two fundamental Lamb waves S0 and A0 in the plate with the hiding partial-thickness notch. The incident pulses were generated below

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the cut-off frequency of the mode A1 to only excite S0 and A0 modes. It has been reported that when the notch width is smaller than the wavelength, the scattering of the Lamb modes is insensitive to the notch width [16], so that only the notch-depth variation was considered in this study. In the theory section, the apparent group velocities of the converted modes were expressed as functions of the notch positions and propagation distances. The proposed apparent group velocity equalities were testified by the two-dimensional finite difference time domain (2D-FDTD) simulations and experiments. In signal processing procedure, a mask filtering approach [8,24] was applied to extract the original and converted modes out of the measured matrix of the Lamb waves recorded at shift transmitter–receiver distances. Finally, frequency-sweeping experiments were performed in the steel plates with respect to notch-depth variation, and the energy percentages of the original and converted modes were used to describe the mode conversion, together with the notch-depth evaluation.

2. Lamb waves theory

2.1. Theoretical solutions of the Lamb waves

Theoretically, Lamb waves refer to the strain and stress, propagating along a solid plate (or layer) with the traction-free boundary condition [1]. Classified by the displacements distribution in the propagation direction, there are two categories of Lamb waves, symmetrical and anti-symmetrical modes. The dispersion relation in a plate of thickness h are satisfied by the Rayleigh–Lamb equations [1]

$$\text{Symmetrical modes}:\ (k^2 - q^2)^2\tan(qh/2) + 4k^2pq\tan(ph/2) = 0 \tag{1a}$$

$$\text{Anti-symmetrical modes}:(k^2 - q^2)^2\tan(ph/2) + 4k^2pq\tan(qh/2) = 0 \tag{1b}$$

$$k^2 = \frac{\omega^2}{V_p^2},\quad p^2 = \frac{\omega^2}{V_L^2},\quad q^2 = \frac{\omega^2}{V_T^2}, \tag{1c}$$

where k, ω , V_p, V_L and V_T are the angular wavenumber, angular frequency, phase velocity, and velocities of longitudinal and shear wave, respectively. The Eq. (1) can be numerically solved to obtain the velocity curves. The phase velocity can be regard as the propagation speed of the wave phase of a particular frequency, which also denotes that Lamb waves are dispersive. Another overall velocity of the mode-packet, usually named as group velocity, is also used to describe the Lamb wave propagation [1]

$$V_g(f \cdot h) = \frac{d\omega}{dk} \tag{2}$$

where the $f \cdot h$ is the frequency-thickness product. According to Eqs. (1) and (2), the phase and group velocities of Lamb modes can be numerically determined to obtain the dispersion curves.

2.2. Apparent group velocity of the converted Lamb modes

Although dispersion relationship accurately determines the modes propagation velocities versus $f \cdot h$, Lamb waves application is still a bit troublesome in some cases because of the existence of multimodal packets at any given frequency. In addition, the inherent dispersion of the Lamb modes also impairs the modal amplitude, decreases the signal-to-noise ratio, and spreads the modal overlapping. Interaction of Lamb modes with defects and edges on the waveguide further sophisticates the signal interpretation. Therefore, the narrowband excitation has been widely adopted to generate the non-dispersive guided modes propagating at constant velocities, which highly facilities the mode identification and wave-packet separation. For example the Gaussian-windowed toneburst was employed to suppress the Lamb mode overlap [16].

As shown in Fig. 1, modes S0 and A0 are excited in a plate with an anti-symmetrical notch by a narrowband low-frequency pulse. The vibration propagates from the left side of the plate to the right through the transverse hiding notch. Consequently, besides the original modes S0 and A0, the converted modes $\mathrm{S0_{A0}}$ (which means mode S0 derived or converted from A0) and $\mathrm{A0_{S0}}$ (which means mode A0 derived from S0) can be acquired by the receiver, simultaneously. Assuming it is a narrowband excitation with a small dispersion, the group velocities of A0 and S0 can be regarded as two constants, $V_g(\mathrm{S0})$ and $V_g(\mathrm{A0})$. The distances from the notch to the left transmitter and right receiver are $d1$ and $d2$, respectively. The apparent group delays of the $\mathrm{S0_{A0}}$ and $\mathrm{A0_{S0}}$ are formulated by

$$T(\mathrm{A0_{S0}}) = d1/V_g(\mathrm{S0}) + d2/V_g(\mathrm{A0}) \tag{3a}$$

$$T(\mathrm{S0_{A0}}) = d1/V_g(\mathrm{A0}) + d2/V_g(\mathrm{S0}) \tag{3b}$$

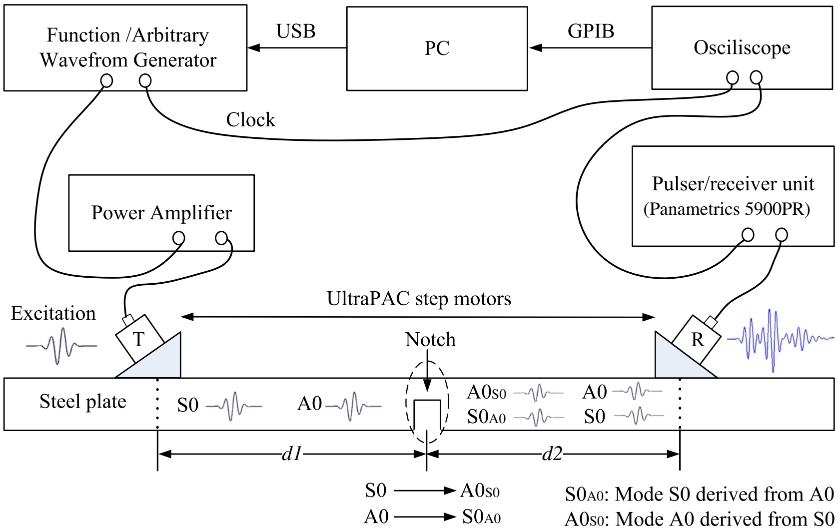

Fig. 1. Schematic of experimental setup and transmitted mode conversion on the notched plate.