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Lamb waves in phononic crystal slabs: Truncated plane parallels to the axis of periodicity

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ABSTRACT

A theoretical study is presented on the propagation properties of Lamb wave modes in phononic crystal slabs consisting of a row or more of parallel square cylinders placed periodically in the host material. The surfaces of the slabs are parallel to the axis of periodicity. The dispersion curves of Lamb wave modes are calculated based on the supercell method. The finite element method is employed to calculate the band structures and the transmission power spectra, which are in good agreement with the results by the supercell method. We also have found that the dispersion curves of Lamb waves are strongly dependent on the crystal termination, which is the position of the cut plane through the square cylinders. There exist complete or incomplete (truncated) layers of square cylinders with the change of the crystal termination. The influence of the crystal termination on the band gaps of Lamb wave modes is analyzed by numerical simulations. The variation of the crystal termination leads to obvious changes in the dispersion curves of the Lamb waves and the widths of the band gaps.

1. Introduction

It may be well-known that elastic vibrations in the periodic arrays called phononic crystals satisfy a frequency band structure. Phononic crystals (PCs) consist of composite materials with elastic coefficients which vary periodically in space. During the past decades, the propagation of elastic waves in PCs has attracted much attention [1,2]. The occurrence of band gaps, where the propagation of acoustic or elastic waves with frequencies within the gaps is forbidden regardless of the direction, suggests numerous technological applications such as acoustic filters, ultrasonic silent blocks, focus lens and so on [3–9]. More recently, the properties of the plate-mode waves in two-dimensional (2D) phononic crystal (PC) slabs have been intensively studied because of their potential practice [10]. Unlike the infinite PCs for bulk waves, which are infinite along three dimensions in real space, the PCs studied for the plate-mode waves are taken to be a finite size system in one direction. Hou et al. have investigated the propagation of elastic wave modes in a two-layer free standing plate composed of a one-dimensional PC thin layer coated on a uniform substrate [11]. Chen et al. have investigated the propagation of Lamb waves in one-dimensional PC plate coated symmetrically with solid loading layers on both sides [12]. It has been shown that Lamb waves can be supported in 2D PC slabs with the slab surfaces perpendicular to the axis of periodicity and the ratio of the thickness of the slabs to the lattice period influences the band gaps of Lamb waves [13–15].

However, another configuration that the surfaces of slabs are parallel to the axis of a row of square cylinders also supports Lamb waves. The PC slabs of this configuration have the potential applications in high-frequency wireless communication and sensing systems and we studied the structure for precast-slab-type construction inspection and other reliable nondestructive evaluation. In this paper, we investigate the characteristics of Lamb waves in PC slabs consisting of a row or more of square cylinders placed periodically in the host material as shown in Fig. 1. This paper is organized as follows. In Section 2, the theory of the supercell method is explained. In Section 3, the geometry model of calculation will be exhibited. The dispersion curves of the conventional structure are calculated by the supercell method and the finite element (FE) method (Comsol Multiphysics 3.5a), which are in good agreement with the transmission power spectra (TPS). The influence of the crystal termination on the band gaps of Lamb wave modes is analyzed by numerical simulations. The variation of the crystal termination leads to obvious changes in the locations and the widths of the band structures. Conclusions drawn in this paper are given in Section 4.

2. Theory

The equation governing the motion of displacement vector $\mathbf{u}(\mathbf{r}, t)$ can be written as

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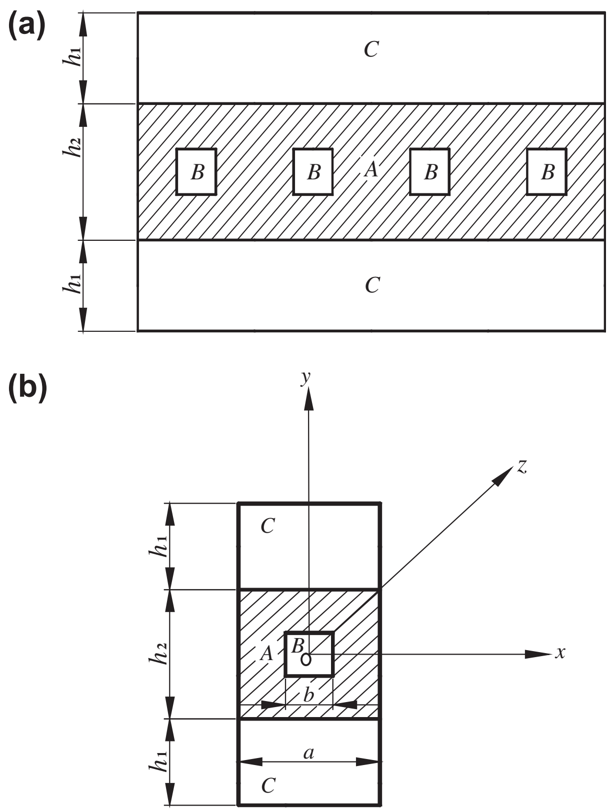

Fig. 1. (a) Auxiliary superstructure for calculations of Lamb wave. (b) Unit cell of (a), two vacuum layers with thickness h_1 are added on their outer surfaces. The h_2 equals to the lattice space a and the side length of the elastic square cylinders (denoted by B) is d.

$$\rho(\mathbf{r})\ddot{u}_p = \partial_q[c_{pqmn}(\mathbf{r})\partial_n u_m] \ (p = 1, 2, 3), \tag{1}$$

where $\mathbf{r}$=($\mathbf{x}$, z)=(x, y, z) is the position vector, $\rho(\mathbf{r})$ and $c_{pqmn}(\mathbf{r})$ are the position-dependent mass density and elastic stiffness tensor, respectively, and the summation convention over repeated indices is assumed in Eq. (1). Due to the spatial periodicity, the material constants, $\rho(\mathbf{r})$ and $c_{pqmn}(\mathbf{r})$, can be expanded in the Fourier series with respect to the 2D reciprocal lattice vectors (RLV), $\mathbf{G} = (G_x, G_y)$, as the follows:

$$\rho(\mathbf{r}) = \sum_{\mathbf{G}} e^{j\mathbf{G}\cdot\mathbf{x}} \rho_{\mathbf{G}} \tag{2}$$

$$c_{pqmn}(\mathbf{r}) = \sum_{\mathbf{G}} e^{j\mathbf{G}\cdot\mathbf{x}} c_{\mathbf{G}}^{pqmn} \tag{3}$$

Utilizing the Bloch theorem and expanding the displacement vector $\mathbf{u}(\mathbf{r}, t)$ in Fourier series, one obtains

$$\mathbf{u}(\mathbf{r}, t) = \sum_{\mathbf{G}} \mathbf{u}_{\mathbf{G}} e^{j(\mathbf{K}+\mathbf{G})\cdot\mathbf{x} - j\omega t} \tag{4}$$

where $\mathbf{k} = (k_x, k_y)$ is a Bloch wave vector and ω is the circular frequency. Note that ρ and c_{pqmn} are independent on the z-direction because of the homogeneity of the system along the cylinder axis. By substituting Eqs. (2)–(4) into Eq. (1), it can be shown that the wave motion polarized in the z-direction, namely the SH wave, decouples from the wave motions polarized in the x- and y-directions, namely, the P and the SV waves. We focus our attentions to the P and the SV waves because it is relatively simple to discuss the SH wave. The Lamb waves can be obtained by the coupling between the P and the SV waves since two vacuum layers (denoted by C in Fig. 1b) with thickness h_1 are used for designing the imaginary periodic system and the stress-free boundary condition for Lamb waves is satisfied.

3. Numerical results

3.1. Geometry and method of calculation

We first briefly introduce the PC slabs, which are composed of unit cells consisting of embedded elastic square cylinders (denoted by B) of side length d and matrix material (denoted by A) with lattice spacing a. The material A and B are the Tungsten and the Vacuum, respectively. The elastic parameters used in the calculations are $C_A^{11} = 502\ GPa$, $C_A^{12} = 199\ GPa$ and $C_A^{44} = 152\ GPa$ and mass density $\rho_A = 19200 Kg/m^3$ for the Tungsten.

Firstly the band structures of the Lamb wave modes were computed using the supercell method, which has been proven in previous works to be an efficient method for obtaining the curves of plate-mode waves in PC slabs. The key point of the supercell method is to design an appropriate auxiliary infinite periodic superstructure in order to apply the Bloch theorem. The filling rate f is defined by $f = (d^2)/(a^2)$ and the thickness of slabs h_2 equals to the lattice spacing, as shown in Fig. 1. The reciprocal lattice vector is $\mathbf{G} = (2\pi N_1/a,\ 2\pi N_2/(h_2 + 2h_1))$, with N_1 and N_2 integers, respectively. In the calculation, $f = 0.09$ and 529 RLV are used to reach good convergence.

Secondly, we have made the calculation of the dispersion curves for PC slabs by using the FE method with the Comsol Multiphysics 3.5a. The calculation is performed by solving the eigenvalue problem of the unit cell. The main step of the calculation will be presented. First, the periodic boundary conditions are applied to the sides of unit cell along the x-direction based on the Bloch's theorem. The periodic boundary conditions in terms of displacement u_p could be defined as $u_p(x + nd, y) = u_p(x, y)\exp[-j(k_x nd)]$, where k_x is the wave vector in the x-direction and n is an arbitrary integer. Moreover, the top and button surfaces are defined free boundary conditions. Second, the 2D cross section of the supercell in the x–y plane is meshed and divided into finite elements. The finite elements are triangles with three nodes that has two degrees of freedoms u_x and u_y. Finally, the FE method transforms the analysis into a generalized elastic eigenvalue problem expressed by the linear equations $[K(k_x) - w^2 M]u = 0$, where $K(k_x)$ and M are the stiffness and mass matrices of the system. The relationship between the angular frequency w and the wave number k_x is inherent in these equations. Then the band structure $w = w(k_x)$ can be built by sweeping k_x through the entire first Brillouin zone. An alternative searching arithmetic is employed to determine the eigenfrequency w.

Fig. 2a shows the low-frequency part of the dispersion curves of the Lamb wave modes with $f = 0.09$, $d = 0.3$ mm, and $a = 1.0$ mm. The horizontal axis is the reduced wave number $k^* = ka/\pi$. As shown in Fig. 2a, the data which is computed by the supercell method is marked with the solid lines, and the points represent the data calculated using the FE method. We can see that the locations and widths of band gaps of the dispersion curves from FE analysis are in good agreement with the results by the supercell method, and we find a band gap (shaded region) which is from 1.372 to 1.745 MHz between the third and the fourth transmission band. The value of the gap width is 0.3724 MHz and the corresponding gap/midgap ratio is approximately 0.2389. In the band gap, all acoustic modes are suppressed.

Finally, in order to further demonstrate the existence of the band gaps for the lower-order modes in the PC slabs, the FE method was employed to calculate the TPS for the finite periodic structure. The TPS of this structure corresponding to Fig. 2a has been performed as shown in Fig. 2b. There is a broad region ranged from 1.375 to 1.752 MHz for which the transmission is less than –