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Study on the step-type circular ring ultrasonic concentrator in radial vibration

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ABSTRACT

In this paper, the plane radial vibration of an isotropic metal thin circular rings is studied and its equivalent circuit model is obtained. Based on the equivalent circuit model, the step-type circular ring concentrator consisting of two metal thin circular rings in radial vibration is analyzed. Its compound equivalent circuit is derived and the resonance frequency equation is obtained. The relationship between the resonance frequency, the radial displacement amplitude magnification and the geometrical dimensions is analyzed. The resonance frequency of the step-type radial concentrator is calculated based on the resonance frequency equation. For comparison, the resonance frequency of the step-type radial concentrator is also obtained by using numerical method. It is illustrated that the resonance frequencies from these two methods are in a good agreement with each other.

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1. Introduction

In the fields of power ultrasonics and underwater sound, ultrasonic concentrators (which are also known as ultrasonic transformers or ultrasonic horns) are widely used in order to focus ultrasonic energy, magnify vibrational displacement amplitude and convert vibrational direction. According to vibrational modes, ultrasonic concentrators can be divided into longitudinal concentrators, torsional concentrators, flexural concentrators, radial concentrators and composite mode concentrators. For ultrasonic concentrators, different vibrating elements are needed. The most widely used vibrating elements include slender or short circular rods with constant or varying cross-section, circular hollow cylinders or rings, circular or rectangular plates, and their combinations. For example, in high power ultrasonics, metal cylinders and cones can be used as the back and front end masses of sandwich piezoelectric ultrasonic transducers; metal rods with varying cross-section can be used as the displacement amplitude transformer in ultrasonic machining and ultrasonic metal and plastic welding; circular or rectangular metal plates may be used as radiating elements of vibrating systems as in ultrasonic levitation and cleaning.

Nowadays longitudinal concentrator is the most widely used one and its design theory is well established [1–4]. The ultrasonic concentrator in radial vibration, which consists of circular disks or rings with varying cross-section, however, has not been studied thoroughly. The vibration of rings and cylindrical shells has been widely studied for over a century. Much of the related work is summarized in references [5,6]. In the last few decades, some new works are reported on the coupled vibration of cylinders, rings and shells [7–11]. For these vibrating elements, vibrational modes include extensional, torsional, flexural vibration and their coupling vibrations. In general cases, the vibration of circular rings is complex, exact analytical solutions are difficult to find. However, when the geometrical dimensions of circular rings satisfy certain conditions, its vibration can be greatly reduced. Kleesattel and Gladwell studied the radial vibration of disk and ring resonators vibrating in radial and torsional modes [12–14].

In this paper, the radial vibration of an isotropic metal thin circular ring is studied; its equivalent circuit is derived. Based on the equivalent circuit model, the radial vibration of a step-type circular ring concentrator is studied. Its equivalent circuit is derived and the resonance frequency equation is obtained. The relationship between the resonance frequency, the radial displacement amplitude magnification and the geometrical dimensions is analyzed.

2. Radial vibration of a metal thin circular ring

A metal thin circular ring in radial vibration is shown in Fig. 1. Its thickness, outer and inner radiuses are L, a and b. In the figure, v_{ra}, v_{rb} and F_{ra}, F_{rb} are radial vibrational velocities and external forces at the outer and inner surfaces. In polar coordinates, the wave equations of a metal circular ring can be obtained as

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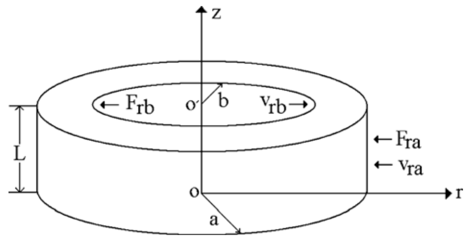

Fig. 1. Geometrical diagram of a metal thin circular ring in radial vibration.

$$\rho\frac{\partial^2\xi_r}{\partial t^2}=\frac{\partial T_r}{\partial r}+\frac{1}{r}\cdot\frac{\partial T_{r\theta}}{\partial\theta}+\frac{\partial T_{rz}}{\partial z}+\frac{T_r-T_\theta}{r},\tag{1}$$

$$\rho\frac{\partial^2\xi_\theta}{\partial t^2}=\frac{\partial T_{r\theta}}{\partial r}+\frac{1}{r}\cdot\frac{\partial T_\theta}{\partial\theta}+\frac{\partial T_{\theta z}}{\partial z}+\frac{2T_{r\theta}}{r},\tag{2}$$

$$\rho\frac{\partial^2\xi_z}{\partial t^2}=\frac{\partial T_{rz}}{\partial r}+\frac{1}{r}\cdot\frac{\partial T_{\theta z}}{\partial\theta}+\frac{\partial T_z}{\partial z}+\frac{T_{rz}}{r}.\tag{3}$$

Here ξ_r, ξ_θ, ξ_z are radial, tangential and axial displacement components, T_r, T_θ, T_z, $T_{r\theta}$, T_{rz}, $T_{\theta z}$ are stress components in the ring. The relationship between the strains and the displacements are expressed as

$$S_r=\frac{\partial\xi_r}{\partial r},\quad S_\theta=\frac{1}{r}\cdot\frac{\partial\xi_\theta}{\partial\theta}+\frac{\xi_r}{r},\quad S_z=\frac{\partial\xi_z}{\partial z},\tag{4}$$

$$S_{r\theta}=\frac{1}{r}\cdot\frac{\partial\xi_r}{\partial\theta}+\frac{\partial\xi_\theta}{\partial r}-\frac{\xi_\theta}{r},\quad S_{\theta z}=\frac{1}{r}\cdot\frac{\partial\xi_z}{\partial\theta}+\frac{\partial\xi_\theta}{\partial z},\quad S_{rz}=\frac{\partial\xi_r}{\partial z}+\frac{\partial\xi_z}{\partial r}.\tag{5}$$

Here S_r, S_θ, S_z, $S_{r\theta}$, $S_{\theta z}$, S_{rz} are strain components. According to Hooke's law, the relationship between strains and stresses are

$$S_r=\frac{1}{E}[T_r-\nu(T_\theta+T_z)],\quad S_\theta=\frac{1}{E}[T_\theta-\nu(T_r+T_z)],\quad S_z=\frac{1}{E}[T_z-\nu(T_\theta+T_r)],\tag{6}$$

$$S_{r\theta}=\frac{T_{r\theta}}{G},\quad S_{rz}=\frac{T_{rz}}{G},\quad S_{\theta z}=\frac{T_{\theta z}}{G}.\tag{7}$$

Here $G=\frac{E}{2(1+\nu)}$ is shearing modulus, E and ν are Young's modulus and Poisson's ratio of the ring material. It is obvious that the vibration of a metal circular ring is a complex coupled one and its analytical solutions are difficult to find. To simplify the analysis of a metal circular ring, it is assumed that the ring is an isotropic thin circular ring. Its thickness is much less than its radius, i.e. $L\ll a$. In this case, we have, $T_z=0$, $T_{zr}=0$, $T_{z\theta}=0$, $T_{r\theta}=0$, $\frac{\partial T_r}{\partial\theta}=\frac{\partial T_\theta}{\partial\theta}=\frac{\partial T_z}{\partial\theta}=\frac{\partial T_{rz}}{\partial\theta}=\frac{\partial T_{r\theta}}{\partial\theta}=\frac{\partial T_{z\theta}}{\partial\theta}=0$, $\xi_z=0$, $\xi_\theta=0$, $\frac{\partial\xi_\theta}{\partial\theta}=0$. The above equations can be reduced to the following forms:

$$\rho\frac{\partial^2\xi_r}{\partial t^2}=\frac{\partial T_r}{\partial r}+\frac{T_r-T_\theta}{r},\tag{8}$$

$$T_r=\frac{E}{1-\nu^2}(S_r+\nu S_\theta),\quad T_\theta=\frac{E}{1-\nu^2}(S_\theta+\nu S_r),\tag{9}$$

$$S_r=\frac{\partial\xi_r}{\partial r},\quad S_\theta=\frac{\xi_r}{r}.\tag{10}$$

Based on these three equations, the wave equation for a metal thin ring in radial vibration can be obtained as

$$\frac{\partial^2\xi_r}{\partial t^2}=V_r^2\left(\frac{\partial^2\xi_r}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial\xi_r}{\partial r}-\frac{\xi_r}{r^2}\right).\tag{11}$$

Here $V_r^2=\frac{E}{\rho(1-\nu^2)}$, V_r is sound speed of radial vibration in a thin metal circular ring. The solution to Eq. (11) is

$$\xi_r=[AJ_1(kr)+BY_1(kr)]e^{j\omega t}.\tag{12}$$

Here $k=\omega/V_r$, ω is angular frequency, $J_1(kr)$ and $Y_1(kr)$ are Bessel functions of order one, A and B are two constants, they can be determined by the boundary conditions of the ring. From Eq. (12), the radial vibrational velocity can be obtained

$$v_r=j\omega[AJ_1(kr)+BY_1(kr)]e^{j\omega t}.\tag{13}$$

From Fig. 1, we have, when $r=a$, $v_r=-v_{ra}$; when $r=b$, $v_r=v_{rb}$. Therefore, the constants A and B can be obtained as

$$B=\frac{1}{j\omega}\cdot\frac{v_{ra}J_1(kb)+v_{rb}J_1(ka)}{J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)}e^{-j\omega t},\tag{14}$$

$$A=-\frac{1}{j\omega}\cdot\frac{v_{ra}Y_1(kb)+v_{rb}Y_1(ka)}{J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)}e^{-j\omega t}.\tag{15}$$

Substituting these two expressions into the expression of the radial stress T_r yields

$$T_r=\frac{Ek}{1-\nu^2}\left\{A\left[J_0(kr)-\frac{(1-\nu)J_1(kr)}{kr}\right]+B\left[Y_0(kr)-\frac{(1-\nu)Y_1(kr)}{kr}\right]\right\}e^{j\omega t}.\tag{16}$$

Using the boundary conditions of $F_r=T_r|_{r=a}\cdot S_{ra}=-F_{ra}$ and $F_r=T_r|_{r=b}\cdot S_{rb}=-F_{rb}$, we have

$$F_{ra}=-\frac{ES_{ra}}{1-\nu^2}\cdot\frac{k}{j\omega[J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)]}\times\{v_{ra}[Y(a)J_1(kb)-J(a)Y_1(kb)]+v_{rb}[Y(a)J_1(ka)-Y_1(ka)J(a)]\},\tag{17}$$

$$F_{rb}=-\frac{ES_{rb}}{1-\nu^2}\cdot\frac{k}{j\omega[J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)]}\times\{v_{ra}[Y(b)J_1(kb)-J(b)Y_1(kb)]+v_{rb}[Y(b)J_1(ka)-Y_1(ka)J(b)]\}.\tag{18}$$

Here $S_{ra}=2\pi aL$, $S_{rb}=2\pi bL$, S_{ra} and S_{rb} are outer and inner surface areas of the metal ring. In Eqs. (17) and (18), $J(a)$, $J(b)$ and $Y(a)$, $Y(b)$ are four introduced functions, their expressions are

$$J(a)=J_0(ka)-\frac{(1-\nu)J_1(ka)}{ka},\quad J(b)=J_0(kb)-\frac{(1-\nu)J_1(kb)}{kb},$$
$$Y(a)=Y_0(ka)-\frac{(1-\nu)Y_1(ka)}{ka},\quad Y(b)=Y_0(kb)-\frac{(1-\nu)Y_1(kb)}{kb}.$$

After some transformations, Eqs. (17) and (18) can be rewritten as

$$\frac{F_{ra}}{jZ_a}=v_{ra}\left[\frac{1-\nu}{ka}+\frac{Y_0(ka)J_1(kb)-J_0(ka)Y_1(kb)}{J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)}\right]+v_{rb}\left[\frac{J_1(ka)Y_0(ka)-J_0(ka)Y_1(ka)}{J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)}\right],\tag{19}$$

$$\frac{F_{rb}}{jZ_b}=v_{ra}\left[\frac{Y_0(kb)J_1(kb)-J_0(kb)Y_1(kb)}{J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)}\right]+v_{rb}\left[\frac{J_1(ka)Y_0(kb)-J_0(kb)Y_1(ka)}{J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)}-\frac{1-\nu}{kb}\right].\tag{20}$$

Here $Z_a=\rho V_r S_{ra}$, $Z_b=\rho V_r S_{rb}$. Eqs. (19) and (20) can be further expressed as the following forms:

$$F_{rb}=(Z_{1m}+Z_{3m})v_{rb}+Z_{3m}v_{ra},\tag{21}$$

$$F_{ra}=(Z_{2m}+Z_{3m})v_{ra}+Z_{3m}v_{rb}.\tag{22}$$

Here Z_{1m}, Z_{2m} and Z_{3m} are three mechanical impedances, their expressions are

$$Z_{1m}=j\frac{2Z_{rb}}{\pi kb[J_1(ka)Y_1(kb)-J_1(kb)Y_1(ka)]}\times\left[\frac{J_1(ka)Y_0(kb)-J_0(kb)Y_1(ka)-J_1(kb)Y_0(kb)+J_0(kb)Y_1(kb)}{J_1(kb)Y_0(kb)-J_0(kb)Y_1(kb)}\right]-j\frac{2Z_{rb}(1-\nu)}{\pi(kb)^2[J_1(kb)Y_0(kb)-J_0(kb)Y_1(kb)]};$$

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