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Mutual conversion of bulk and surface acoustic waves in gratings of finite length on half-infinite substrates. II. FE analysis of bulk wave generation

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ABSTRACT

The paper studies numerically the bulk acoustic wave generation by the surface acoustic wave propagating across a grating created on the surface of an elastically anisotropic half-infinite substrate. The computations are fully based on the finite element method. Applying the discrete Fourier transformation to the displacement field found inside the substrate and using an orthogonality relation valid for plane modes we determine separately the spacial spectrum of the quasi longitudinal and the quasi transverse bulk waves, that is, the dependence of the amplitudes of these waves on the tangential component of the wave vector. The dependence is investigated of the central spectral peak height and shape on the frequency of the incident surface wave as well as on the thickness, the width, and the number of strips forming the grating. In particular, it is found that under certain conditions the central peak can be approximated fairly precisely by the central peak of a sinc-function describing the spectrum of the bounded acoustic beam of rectangular shape and of width equal to the length of the grating.

1. Introduction

The propagation of the surface acoustic wave (SAW) across surface inhomogeneities is accompanied by the irradiation of bulk acoustic waves (BAWs). On the one hand, the SAW–BAW conversion is an undesirable phenomenon when the grating plays the role of a reflector or an interdigital transducer [1,2]. On the other hand, this conversion can be useful for transmitting an acoustic signal from one surface to the opposite one or for receiving the surface wave signal by a bulk wave transducer [3].

The scattering of surface waves into bulk waves caused by single grooves and periodically corrugated areas in isotropic media has been intensively studied by different approximate analytic techniques. In particular, Ref. [4] theoretically investigates the Rayleigh attenuation because of the scattering from a weak periodic roughness on the basis of the so-called Rayleigh hypothesis and using the lowest approximation of a boundary perturbation technique. Within the frame of the same approximations the generation of bulk waves by the Rayleigh wave incident onto a single groove is analyzed in Refs. [5,6]. The results of Ref. [5] were generalized in Ref. [7] to the case of infinite and semi-infinite periodic gratings.

In Ref. [8], still in the lowest order of the perturbation theory but employing an impedance approach, a thorough consideration is performed of the mode conversion accompanying the Rayleigh wave scattering from single inhomogeneities. The SAW–BAW conversion in grating areas of sinusoidal and non-sinusoidal profiles is also investigated with the help of the coupling-of-mode (COM) theory in [8] as well as in [9–11]. The dispersion equation for the Rayleigh wave on the periodically corrugated surface and the analysis of its attenuation due to conversion into bulk waves is carried in [12,13] using the extinction theorem form of Green's theorem.

Besides, the plane wave expansion method has been employed to study the propagation of surface waves across a grating of strips on isotropic film-substrate structures [14]. Unlike the COM-theory, more than two spatial harmonics are included in the expansions. Note that Ref. [14] takes into consideration only the mass loading effect.

Surface waves in piezoelectric crystals are sensitive to changes in the electrical state of the surface. The conversion of the surface wave into bulk waves due to a metallic grating on a piezoelectric substrate is investigated numerically in [15] by the Green's function method in the spirit of the theory developed in [16]. Accordingly, this approach takes into account only the scattering because of the metallization and disregards completely the influence of the mechanical loading.

In our previous paper [17] the generation of surface waves by bulk waves is investigated using the finite element method (FEM). The present paper studies on the basis of FEM the converse phenomenon, specifically, the irradiation of bulk waves by the surface wave travelling through a grating of finite length on the

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surface of an elastically anisotropic half-infinite substrate. FEM allows one to overstep limits of some approximations made in previous works. For instance, such simulations properly take into account the effect of the grating ends as well as the effect of mechanical perturbation of the surface. Besides, there is no need to suppose that the wave coupling within a single period of the grating is weak.

As in [17], the computational domain is truncated by the perfectly matched layer (PML).

2. Statement of the problem and computation procedure

We chose again the substrate of symmetry $m3m$ (silicon) and consider the same geometry as in [17], i.e., the x- and z-axes of the coordinate system are parallel to the [100] and [001] directions, respectively. The substrate occupies the half-space $z < 0$. Infinitely long strips, or grooves, are parallel to the y-axis. The surface wave of frequency $\omega = 2\pi f$ propagates along the x-axis. The point $x = z = 0$ is in the middle of the grating.

The wave field of the incident surface wave $\mathbf{u}_{saw}$ is a known function of coordinates. It is required to find the scattered field $\mathbf{u}_{sc}$ and afterwards to calculate the spatial Fourier spectrum of longitudinal and transverse bulk waves, that is, the amplitudes of these waves as functions of the x-component k_x of the wave vector.

Unlike BAW–SAW conversion [17], the reverse conversion is asymmetric with respect to the middle point of the grating. Nevertheless the simulations can also be performed within the domain shown in Fig. 1. To this end, the field $\mathbf{u}_{saw}$ of the incident surface wave should be decomposed into two parts $\mathbf{u}_{saw} = \mathbf{u}_{saw1} + \mathbf{u}_{saw2}$ and the computations are executed twice, where $\mathbf{u}_{saw1}$ and then $\mathbf{u}_{saw2}$ play the role of source fields.

Let us introduce two harmonic fields:

$$\mathbf{u}_{saw1}(x,z) = 0.5\left[\mathbf{u}_{saw} + \mathbf{u}^*_{saw}\right],\quad \mathbf{u}_{saw2}(x,z) = 0.5\left[\mathbf{u}_{saw} - \mathbf{u}^*_{saw}\right]/i, \tag{1}$$

where the asterisk stands for complex conjugation and the common factor $\exp(-i\omega t)$ is omitted. It appears that the x- and z-components of $\mathbf{u}_{saw1,2}$ has the form (see Appendix A)

$$u_{saw1,x} = F(z)\sin(kx),\ \ u_{saw1,z} = G(z)\cos(kx),\quad u_{saw2,x} = -F(z)\cos(kx),\ \ u_{saw2,z} = G(z)\sin(kx). \tag{2}$$

The dependence of $\mathbf{u}_{saw1,2}$ on the z-coordinate specified by the functions $F(z)$ and $G(z)$ is not of importance.

We infer from Eq. (2) that $u_{saw1,x}$ and $u_{saw1,z}$ are the x-odd and the x-even functions, respectively, while $u_{saw2,x}$ and $u_{saw2,z}$ are the x-even and the x-odd functions, respectively. Hence, the scattered fields produced by $\mathbf{u}_{saw1}$ and $\mathbf{u}_{saw2}$ in the domain $x \geqslant 0$, $z < 0$ (Fig. 1) should fulfil the boundary conditions $u_1 = \sigma_{13} = 0$ and $u_3 = \sigma_{11} = 0$, respectively, at $x = 0$ (see [17,18]). Once $\mathbf{u}_{sc1,2}$ are found, the total scattered field is determined as follows:

$$u_{sc,x} = \begin{cases} u_{sc1,x}(x,z) + iu_{sc2,x}(x,z), & x \geqslant 0, \\ -u_{sc1,x}(-x,z) + iu_{sc2,x}(-x,z), & x < 0, \end{cases} \tag{3}$$

$$u_{sc,z} = \begin{cases} u_{sc1,z}(x,z) + iu_{sc2,z}(x,z), & x \geqslant 0, \\ u_{sc1,z}(-x,z) - iu_{sc2,z}(-x,z), & x < 0. \end{cases} \tag{4}$$

The procedure of computing $\mathbf{u}_{sc1,2}$ is similar to that described in [17], see the discussion around Eqs. (3)–(5). The field $\mathbf{u}_0$ is replaced with $\mathbf{u}_{saw1,2}$ and, accordingly, the force $\mathbf{f}$ is the force created by $\mathbf{u}_{saw1,2}$ on the groove edges.

The direct calculation of the spacial spectrum of $\mathbf{u}_{sc}$ by discrete Fourier transform (DFT) with respect to the x-coordinate implies the use of a large horizontal length of the computational domain (Fig. 1) for the step in the reciprocal space to be reasonably small. In order to reduce the size of the domain we carry out the FEM computations within an area of horizontal size $L_{FEM} \approx (55\text{–}70)\lambda$ and then apply DFT to the function

$$\mathbf{u}'_{sc}(x,-z_s) = \begin{cases} \mathbf{u}_{sc}(x,-z_s) - \mathbf{u}_{sc,saw}(x,-z_s), & |x| \leqslant L_{FEM}, \\ 0, \quad L_{FEM} < |x| < L_{DFT}, \end{cases} \tag{5}$$

where $L_{DFT} = 650\lambda$, the functions in Eq. (5) are calculated at a distance z_s below the surface, $\mathbf{u}_{sc,saw}(x,-z_s)$ is the field of the two scattered surface waves travelling in the positive and the negative directions of the x-axis. The amplitudes of these SAWs are found by the same method as in [17] (see also [19,20]). The distance z_s is equal to 1.5λ for x in $|\mathbf{u}_{sc,saw}(x,-z_s)|$ to be sufficiently small, since it appears that the amplitude of the surface waves generated on the surface is far larger than the amplitudes of the bulk waves (see Appendix B). Note that $|\mathbf{u}'_{sc}(x,-z_s)|$ are small in fact at $x > L_{DFT}$ (see Fig. 2b).

Knowing the Fourier transform $\mathbf{v}'_{sc}(k_n)$, $k_n = 2\pi n/L_{DFT}$, $n = 0, \pm1, \pm2, \ldots$, of $\mathbf{u}'_{sc}$ (5), we eventually obtain the Fourier transform k_n in $\mathbf{v}_{sc}(k_n)$ of the full scattered field by adding to $\mathbf{v}'_{sc}(k_n)$ the contributions $C_\gamma(k_n \pm k_{saw})^{-1}$ originating from the singularities of the Green function corresponding to the SAWs on the flat surface. The numerical factors C_γ are found by evaluating the appropriate displacement components of the surface wave at $z = -z_s$ with the help of the method discussed in [19,20].

Further, the Fourier transform $\mathbf{t}_{sc}(k_n)$ of the traction produced by $\mathbf{u}_{sc}$ on the plane $z = -z_s$ is found. In addition, an advantage can be taken of an orthogonality relation upon the polarization vector $\mathbf{A}_\alpha$ and the traction $\mathbf{L}_\alpha$ of plane mode solutions to the wave equation in the half-infinite substrate, given frequency and the wave number k_x along the surface: $\mathbf{A}_\alpha\mathbf{L}_\beta + \mathbf{A}_\beta\mathbf{L}_\alpha = \delta_{\alpha\beta}$, where the subscripts α, β identify plane modes. This relation follows from the properties of the 6×6 Stroh matrix, of which the eigenvectors ξ_α, $\alpha = 1, \ldots, 6$, are 6×1 columns $\xi_\alpha = (\mathbf{A}_\alpha, \mathbf{L}_\alpha)^t$, see, e.g., [21–23] (as distinct from [21–23], here k_x is included in the definition of $\mathbf{L}_\alpha$). By this way we eventually calculate the amplitude $b_\alpha(k_n)$ at $z = 0$ of a plane mode α involved in the scattered field and corresponding to $k_x = k_n$:

$$b_\alpha(k_n) = [\mathbf{v}_{sc}\mathbf{L}_\alpha + \mathbf{t}_{sc}\mathbf{A}_\alpha]e^{ik_n p_\alpha z_s}/|\mathbf{A}_\alpha|, \tag{6}$$

where p_α is the cotangent of the propagation angle of the mode α.

The fact that this method is of acceptable precision can be checked by comparing the FEM results with the results of "non-FEM" computations for the case where the displacements in the flat substrate are produced by 2D point harmonic sources from the surface. For example, let alike unit forces directed along the x-axis be applied to the substrate at 20 points spaced by a distance

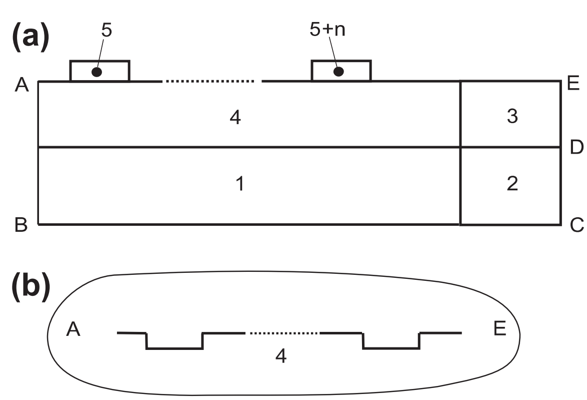

Fig. 1. Computational domain; (a) - grating of strips, (b) - grating of grooves. 1-3 – PML, 4 – substrate, from 5 till (5 + n) – strips. Lines AB and AE correspond to $x = 0$ and $z = 0$, respectively.