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An accurate analysis of the radiation characteristics of a plane piston transducer with phase apodization for focusing

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ABSTRACT

A plane piston transducer can be focused by a continuous variation of the excitation signal phase (or time delay) over the transducer surface. Prior analyses of this scheme used the Fresnel approximation, thereby limiting the validity. Using the angular spectrum method, an accurate radiation model of such a transducer has been developed that includes amplitude and phase apodization. The derivation includes the effects of diffraction and evanescent waves without using the Fresnel approximation. Moreover, this model develops insights into radiated field characteristics, including: (a) the spatial frequency bandwidth is constant over axial depth, suggesting that spatial resolution can be improved away from the focus; (b) the phase of the angular spectrum determines the spatial resolution for a given transducer configuration—a constant phase is optimal on any observation plane; (c) focusing can significantly increase the spatial frequency bandwidth; (d) the velocity potential on a plane parallel to the transducer is the Hankel convolution of the transducer surface velocity with the Green's function; and (e) evanescent waves decay both with increasing spatial frequency and axial depth. The analytical model and associated insights enhance understanding of the radiated field characteristics, which can be of value in the development of signal processing techniques for image enhancement.

1. Introduction

While there is a thorough understanding of the radiated field arising from uniformly excited plane piston transducers [1–5], the focused equivalent, i.e., one that allows a continuous change in phase (or the delay) over the plane piston surface, is incomplete [6,7]. Moreover, it seems that prior work on such a transducer assumed the Fresnel approximation, thereby limiting the analysis. As will be shown, for transducers with small F-numbers this approximation can introduce significant errors.

Most early analytical models of the uniformly excited plane piston transducer used either the Rayleigh integral, the King integral [8] or the Schoch solution [9]. The King integral can be considered a precursor to the angular spectrum method, while the Schoch solution converts the Rayleigh integral solution into a line integral calculated over the transducer surface. However, solutions using these methods have been limited to the uniform or apodized, plane piston transducer without any phase change (e.g., [2,4,10]). Angular spectrum analytical solutions have also been developed for various transducer geometries [11–15]. But, for the cases where phasing was included, the Fresnel approximations were used. Our analysis of such a transducer, hereinafter referred to as a focused plane piston transducer, avoids this restriction.

Focusing can also be achieved by a spherically-shaped concave piston [16–20]. However, unless an annular array structure is used, which is generally limited to relatively few array elements [21,22], such a configuration is limited to a single focal depth.

Using the angular spectrum approach, we present an analysis of the apodized focused plane piston transducer shown in Fig. 1, and from this, identify new features of the radiated field that could be of value for ultrasound image enhancement. Part of our investigation includes the use of the spatial sensitivity function (SSF) introduced by Zemp et al. [23,24]. The SSF provides an alternative approach to the traditional method of interpreting the impulse response as a point spread function (PSF). The PSF, $h(t|\mathbf{r}_1)$, describes the response at $\mathbf{r}$ as a function of time t when the transducer is excited by an impulse of velocity. Thus, the SSF $h(\mathbf{r}|t_1)$ describes the spatial distribution of the field at a given instant of time. As illustrated in Fig. 2, the PSF and SSF are closely related, each providing a different interpretation of the field caused by a transducer velocity impulse. Zemp and Insana [24] analyzed the properties of the PSF and SSF for a linear array transducer by making several important assumptions. To identify insights that may be masked by more complicated configurations, our analysis is for a much simpler geometry and thereby avoids making any significant assumptions.

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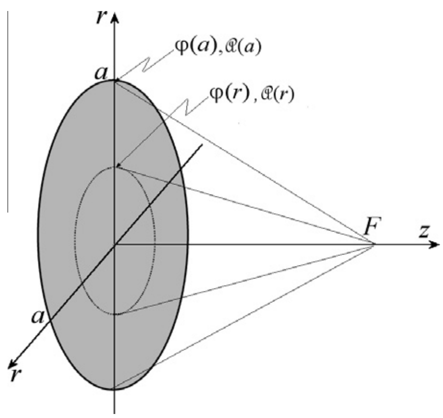

Fig. 1. Sketch of a focused and apodized plane piston with a focal point *F*, showing the cylindrical geometry used in the analysis. It is assumed that the amplitude $\mathcal{A}$ and phase φ of the velocity vary over the piston surface to minimize diffraction effects and to produce focusing.

In summary, our objectives are: (a) to analytically derive equations describing the angular spectrum and velocity potential of the finite geometry, apodized focused plane piston transducer; (b) to simulate and analyze the model equations in order to identify and characterize new radiation characteristics; and (c) to simulate and analyze the impulse response in terms of the PSF and SSF.

2. Theoretical model

2.1. Angular spectrum method

The angular spectrum method enables the field on an observation plane to be predicted from the wave distribution on a parallel plane. Our analysis assumes linear propagation in a homogeneous, inviscid medium and uses cylindrical coordinates (r,z). The following four steps are needed to arrive at the velocity impulse response [25]:

1. Obtain the angular spectrum on the transducer plane ($z = 0$) by performing the zero-order Hankel transform of the normal component of the surface velocity v_z

$$S(k_r:0,\omega) = \frac{-2\pi j}{\sqrt{k^2-k_r^2}}\int_0^\infty v_z(r,0:\omega)J_0(rk_r)r\,dr, \tag{1}$$

is the wave speed, k_r is the spatial frequency, and $J_0(\cdot)$ is the cylindrical Bessel function of the first kind and zero order. The above equation can also be written as,

$$S(k_r:0,\omega) = \frac{-j}{\sqrt{k^2-k_r^2}}H[v_z(r,0:\omega)],$$

where $H[.]$ denotes the Hankel transform defined by Erdelyi [26]

$$H[g(r)] = 2\pi\int_0^\infty g(r)J_0(rk_r)r\,dr = G(k_r),$$

whose inverse is

$$H^{-1}[G(k_r)] = \frac{1}{2\pi}\int_0^\infty G(k_r)J_0(rk_r)k_r\,dk_r = g(r).$$

2. Obtain the angular spectrum on an observation plane at $z > 0$ by using:

$$S(k_r:z,\omega) = S(k_r:0,\omega)\left(e^{-jz\sqrt{k^2-k_r^2}}\Big|_{k_r<k} + e^{-z\sqrt{k_r^2-k^2}}\Big|_{k_r>k}\right), \tag{2}$$

where the homogeneous and evanescent components correspond to $k_r < k$ and $k_r > k$, respectively.

3. Obtain the velocity potential on the observation plane z by taking the inverse Hankel transform

$$\Phi(r,z:\omega) = \frac{1}{2\pi}\int_0^\infty S(k_r:z,\omega)J_0(rk_r)k_r\,dk_r. \tag{3}$$

4. If the transducer temporal excitation velocity is an impulse, then the velocity impulse response $h(r,z:t)$ is given by the inverse temporal Fourier transform $F^{-1}[\cdot]$ of the velocity potential,

$$h(r,z:t) = F^{-1}\left[\frac{1}{2\pi}\int_0^\infty S_\delta(k_r:z,\omega)J_0(rk_r)k_r\,dk_r\right].$$

2.2. Transducer velocity

For the plane piston transducer shown in Fig. 1 having a radius a with Gaussian apodization and focusing, the normal component of the surface velocity distribution is given by (see p. 110,429 in [25])

$$v_z(r,0:\omega) = \mathrm{circ}\left(\frac{r}{a}\right)e^{\frac{-r^2}{2\sigma^2}}e^{jk\left(\sqrt{F^2+r^2}-\sqrt{F^2+a^2}\right)}, \tag{4}$$

where $\mathrm{circ}\left(\frac{r}{a}\right) = \begin{cases}1 & r \leqslant a\\ 0 & r > a\end{cases}$, Gaussian apodization is represented by the first exponential term with a standard deviation of σ, and focusing at an axial depth of F is described by the second exponential term.

It is important to note several characteristics of the transducer velocity equation:

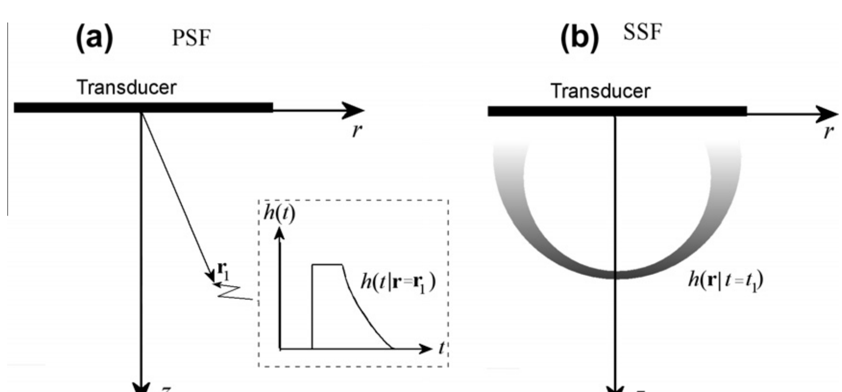

Fig. 2. Sketch illustrating the point spread function (PSF) and the spatial sensitivity function (SSF). (a) The PSF identifies the impulse response over time for a single spatial location. (b) The SSF describes the spatial distribution of the wave at a single point in time.