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A feasibility study of the use of bounded beams resembling the shape of evanescent and inhomogeneous waves

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ABSTRACT

Plane waves are solutions of the visco-elastic wave equation. Their wave vector can be real for homogeneous plane waves or complex for inhomogeneous and evanescent plane waves. Although interesting from a theoretical point of view, complex wave vectors normally only emerge naturally when propagation or scattering is studied of sound under the appearance of damping effects. Because of the particular behavior of inhomogeneous and evanescent waves and their estimated efficiency for surface wave generation, bounded beams, experimentally mimicking their infinite counterparts similar to (wide) Gaussian beams imitating infinite harmonic plane waves, are of special interest in this report. The study describes the behavior of bounded inhomogeneous and bounded evanescent waves in terms of amplitude and phase distribution as well as energy flow direction. The outcome is of importance to the applicability of bounded inhomogeneous ultrasonic waves for nondestructive testing.

1. Introduction

The concept of inhomogeneous waves and evanescent waves (henceforth collectively named 'inhomogeneous waves') in acoustics and ultrasonics originates from the 1960s when they were first considered by Henry Cooper [1,2]. Still, inhomogeneous waves were considered as artifacts until different researchers investigated their properties and significance in real situations encountered in acoustics and in ultrasonics in the 1980s and thereafter [3–24]. Their scattering was studied, leading to a generalized Snell–Descartes law, their damping was studied, but also their propagation properties such as phase velocity, through a generalized dispersion equation, and energy flow. Later, when bounded versions of inhomogeneous waves were generated experimentally, it was found that they could be simulated by means of the Fourier decomposition [19] and also by means of a decomposition into infinite inhomogeneous waves [20] based on the Laplace transformation. An extensive overview of inhomogeneous waves and their history until 2005 was published recently [3].

2. Homogeneous and inhomogeneous harmonic plane waves

Plane waves are solutions of the visco-elastic wave equation. Essentially, normalized harmonic plane waves in liquids are described by a particle velocity potential

$$\exp i(\mathbf{k} \bullet \mathbf{r} - \omega t) \tag{1}$$

in which $\mathbf{k}$ is the wave vector, $\mathbf{r}$ is the position, ω is the angular frequency and t is time.

Being a solution of the wave equation for lossless liquids requires $\mathbf{k}$ to fulfill the dispersion equation

$$\mathbf{k} \bullet \mathbf{k} = \left(\frac{\omega}{v} - i\alpha_0\right)^2 \tag{2}$$

In general $\mathbf{k}$ is complex and therefore, in lossless liquids

$$\mathbf{k} = \mathbf{k}_{real} + i\alpha\mathbf{e}_\alpha - i\beta\mathbf{e}_\beta, \quad \mathbf{e}_\beta \perp \mathbf{k}_{real}, \mathbf{e}_\alpha \parallel \mathbf{k}_{real} \tag{3}$$

The vector $\boldsymbol{\beta}$ is called the inhomogeneity vector, α is the damping of the wave, whereas α_0 is the visco-elastic damping of the material (also called 'intrinsic damping') through which the wave propagates.

Entering (3) in (2) results in two equations, namely

$$\begin{aligned} k_{real}^2 - \beta^2 - \alpha^2 &= \frac{\omega^2}{v^2} - \alpha_0^2 \\ k_{real}\alpha &= -\frac{\omega}{v}\alpha_0 \end{aligned} \tag{4}$$

Homogeneous (plane) waves are characterized by $\beta = 0$ and $\alpha = 0$, damped (plane) waves have $\beta = 0$ and $\alpha \neq 0$. Evanescent (plane) waves have $\beta \neq 0$ and $\alpha = 0$ whereas inhomogeneous (plane) waves have $\beta \neq 0$ and $\alpha \neq 0$. Often evanescent (plane) waves and inhomogeneous (plane) waves are commonly named 'inhomogeneous waves' and sometimes also 'heterogeneous waves'; in this paper we stick to the term 'inhomogeneous waves'.

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The value of β describes the inhomogeneity related to the exponentially decaying amplitude along the wave front, i.e. perpendicular to the phase propagation direction. It is clear from the second equation in (4) that α only exists when α_0 differs from zero. In the following Sections 3–5 we will consider situations when α_0 equals zero, whereas in Section 6 damping, i.e. α_0 different from zero, will be studied.

3. Gaussian beams

A Gaussian beam is defined as a beam resulting from a boundary on which the phase is constant and the amplitude has a Gaussian profile. Upon propagation, the phase is disturbed and tends the beam to spread. This is also called natural sound diffraction. The amplitude remains Gaussian in planes parallel to the boundary. At the boundary, determined by $z = 0$, the Gaussian beam velocity potential is described by

$$\exp(-x^2/W^2) \tag{5}$$

W is the Gaussian beam width. This value is arbitrarily chosen as $W = 0.2$ cm in this paper. It is well known that a beam spreads more when W is small and stays longer parallel when W is large. The frequency used in this paper is 1 MHz. The acoustic medium is considered to be water, resulting in a velocity of sound of 1480 m/s.

It is easily possible to calculate and to represent an entire Gaussian beam in a figure. However, because features of Gaussian beams and their natural diffraction behavior are well-known we limit ourselves to the beam profile at four distances from the boundary $z = 0$. This will allow us to compare with further results for other beam types. The output is given in Fig. 1. The figures are obtained applying a Fourier decomposition of the bounded beam into homogeneous plane waves. Every involved plane wave has an amplitude and phase determined by the Fourier decomposition, whereas their wave vectors are determined in the direction parallel to the boundary (x-direction) by the Fourier decomposition and in the perpendicular direction (z-direction) by the dispersion equation as given by (4). Note that the property of Gaussian amplitude distribution is preserved to a large extent during propagation of sound. Though not represented in this paper, it is also known that the phase near the center of the Gaussian beam is to a large extent in accordance with that of a homogeneous plane wave.

4. Bounded inhomogeneous waves

A bounded inhomogeneous wave is defined similar to a Gaussian beam except that the profile resembles that of an inhomogeneous wave. The exact formulation would depend on the transducer used. Nevertheless, an acceptable description is given by

$$\exp(-\beta x)\exp(-x^{10}/(1.6W)^{10}) \tag{6}$$

In what follows an arbitrarily chosen value of $\beta = -800\ \mathrm{m}^{-1}$ is used. The feature explained in this paper are however also noticeable for other values of β.

The beam profile as described by (6) is depicted in Fig. 2 as a solid line and is compared with the profile of its corresponding infinite inhomogeneous wave $\exp(-\beta x)$ given by a dashed line.

The same procedure is followed as for the Gaussian beam propagation. The result for the propagation pattern of the bounded inhomogeneous wave is shown in Fig. 3 by means of a surface plot (gray scale indicates the amplitude at every position) and in Fig. 4 by means of profiles at different distances from the boundary $z = 0$ similar to Fig. 1.

Note that the amplitude tends to bend to one direction, a property very different from a Gaussian beam. This is particularly visible in Fig. 3 but also in the dashed plot of Fig. 4 at 0.5 cm. Up to around 0.5 cm, the shape is still to a large extent in agreement with that of an infinite inhomogeneous wave and we also found that the in this area the phase is also rather constant parallel to the boundary. Further than 0.5 cm an interesting beam pattern appears that however does certainly not resemble that anymore of an inhomogeneous wave. This is clear in Fig. 4 for plots corresponding to distances further away than 0.5 cm from the boundary. For the given example represented by Figs. 3 and 4 a validity area of around 0.5 cm is therefore noticeable. Out of that area the beam does not resemble an inhomogeneous wave anymore. This is an important feature that must be taken into account when experiments are

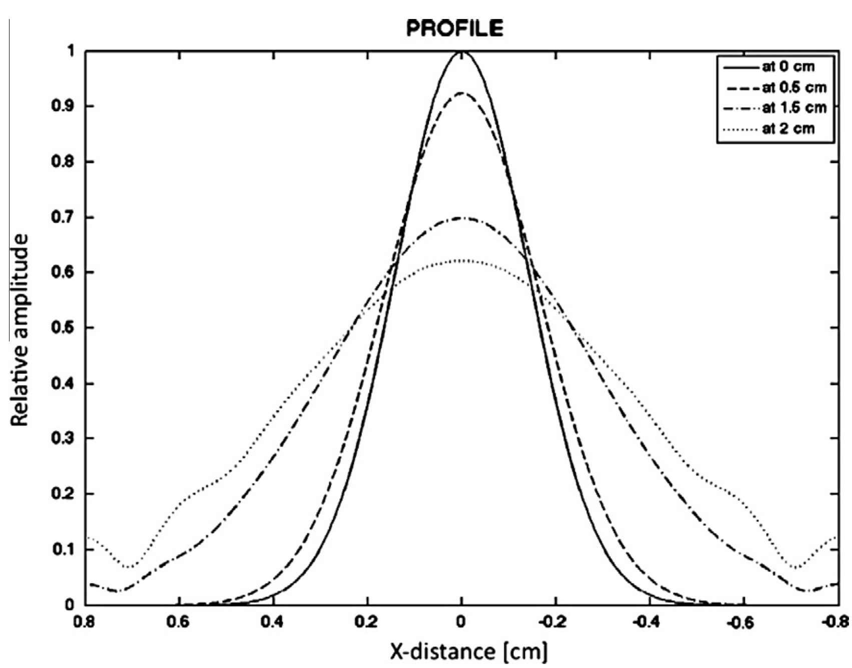

Fig. 1. The amplitude profile of a Gaussian beam at different distances along the path of propagation, for a Gaussian beam width $W = 0.2$ cm.