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Wave propagation of functionally graded material plates in thermal environments

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ABSTRACT

The wave propagation of an infinite functionally graded plate in thermal environments is studied using the higher-order shear deformation plate theory. The thermal effects and temperature-dependent material properties are both taken into account. The temperature field considered is assumed to be a uniform distribution over the plate surface and varied in the thickness direction only. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. Considering the effects of transverse shear deformation and rotary inertia, the governing equations of the wave propagation in the functionally graded plate are derived by using the Hamilton's principle. The analytic dispersion relation of the functionally graded plate is obtained by solving an eigenvalue problem. Numerical examples show that the characteristics of wave propagation in the functionally graded plate are relates to the volume fraction index and thermal environment of the functionally graded plate. The influences of the volume fraction distributions and temperature on wave propagation of functionally graded plate are discussed in detail. The results carried out can be used in the ultrasonic inspection techniques and structural health monitoring.

1. Introduction

The concept of functionally graded materials (FGMs) were the first introduced in 1984 by a group of material scientists in Japan, as ultrahigh temperature resistant materials for aircraft, space vehicles and other engineering applications. Functionally graded materials (FGMs) are new composite materials in which the micro-structural details are spatially varied through non-uniform distribution of the reinforcement phase. This is achieved by using reinforcement with different properties, sizes and shapes, as well as by interchanging the role of reinforcement and matrix phase in a continuous manner. The result is a microstructure that produces continuous or smooth change on thermal and mechanical properties at the macroscopic or continuum level [1,2]. Now, FGMs are developed for general use as structural components in extremely high temperature environments. Therefore, it is important to study the wave propagation of functionally graded materials structures in terms of non-destructive evaluation and material characterization.

The study of the wave propagation in the functionally graded materials structures has received much attention from various researchers. Chen et al. [3] studied the dispersion behavior of waves in a functionally graded plate with material properties varying along the thickness direction. Han and Liu [4] investigated SH waves in FGM plates, where the material property variation was assumed to be a piecewise quadratic function in the thickness direction. Li et al. [5] used the WKB method to investigate the features of Love waves in a layered functionally graded piezoelectric. Chiu and Erdogan [6] studied the one-dimensional wave propagation in a functionally graded elastic medium. Zhang and Batra [7] used the modified smoothed particle hydrodynamics (MSPH) method to study the propagation of waves in functionally graded materials. Han et al. [8] proposed an analytical-numerical method for analyzing the wave characteristics in FGM cylinders. Han et al. [9] also proposed a numerical method to study the transient wave in FGM plates excited by impact loads. Considering the thermal effects, Chakraborty and Gopalakrishnan [10] used the spectral finite element method to analyze the wave propagation behavior in a functionally graded beam subjected to high frequency impulse loading based on the first-order shear deformation theory, where the material properties are temperature-independent, and graded in the thickness direction only. Chakraborty and Gopalakrishnan [11] studied the wave propagation behavior in a functionally graded beam subjected to high frequency loading by using a new beam finite element method based on the first-order shear deformation theory. Bahtin and Eslani [12] analyzed the coupled thermoelastic response of a functionally graded circular cylindrical shell, where the material properties were thickness direction according to a volume fraction power law distribution only. Sun

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and Luo [13,14] investigated the wave propagation and transient response of a FGM plate under a point impact load. Sun and Luo [15] also studied the wave propagation and dynamic response of rectangular functionally graded material plates with completed clamped supports under impulsive load. Yu et al. [16] used the Legendre orthogonal polynomial series expansion approach to investigate characteristics of guided waves in graded spherical curved plates. Elmaimouni et al. [17] proposed a numerical method to analyze the guided wave propagation in an infinite cylinder composed of functionally graded materials (FGM). Yu et al. [18] investigated the propagation of thermoelastic waves in FGM plate in the context of the Green–Naghdi (GN) generalized thermoelastic theory, where the material property variation was assumed to vary in the direction of the thickness only. Yu et al. [19] also studied guided thermoelastic waves in FGM plates in the context of the Green–Lindsay (GL) generalized thermoelastic theories, but they did not take into account temperature-dependent material properties.

In previous works, the studies on the wave propagation of functionally graded materials structures did not consider the thermal effects or considered the thermal effects but did not take into account temperature-dependent material properties. In this paper, considering the thermal effects and temperature-dependent material properties, the wave propagation of an infinite functionally graded plate is studied using the higher-order shear deformation plate theory. The temperature field is assumed to be constant in the plane and only varies in the thickness of the plate. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The governing equations of the wave propagation in the functionally graded plate are derived by using the Hamilton's principle, which the effects of shear deformation and the inertia rotation are taken into account. The analytic dispersion relations of the functionally graded plate are obtained by solving an eigenvalue problem. The dispersion, phase velocity and group velocity curves of the wave propagation in the functionally graded plate in thermal environments are plotted. The characteristics of wave propagation of the functionally graded plate are described in detail. The influences of the volume fraction index N and temperature on the dispersion, phase velocity and group velocity of the wave propagation in the functionally graded plate are clearly discussed. The results carried out can be used in the ultrasonic inspection techniques and structural health monitoring.

2. Properties of the FGM constituent materials

An FGM plate of thickness h is considered here. The materials in top and bottom surfaces of the plate are ceramic and metal, respectively. The material properties P of FGMs are a function of the material properties and volume fractions of the all constituent materials which can be expressed as [20]

$$P = P_t V_c + P_b V_m \tag{1}$$

in which P_t and P_b denote the temperature-dependent properties of the top and bottom surfaces of the plate, respectively, and may be expressed as a function of temperature [21]

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{2}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients showing the temperature-dependency in material properties and are unique to the constituent materials, T (in K) is the environment temperature.

In Eq. (1), V_c and V_m are the ceramic and metal volume fractions, respectively. The sum of volume fractions of the all constituent materials must be unity as follow

$$V_c + V_m = 1 \tag{3}$$

It is assumed that the material composition in an FGM plate varies continuously along the thickness direction only and distribute according to a power law as [22]

$$V_c = \left(\frac{2z+h}{2h}\right)^N \tag{4}$$

where N denotes the power law index which takes values greater than or equal to zero. When $N = 0$, the plate is homogeneous.

It is assumed that the effective Young's modulus E, Poisson's ratio ν and thermal expansion coefficient α of an FGM plate are temperature-dependent, whereas the mass density ρ and thermal conductivity λ of an FGM plate are independent of the temperature. Using Eqs. (1), (2) and (4), they can be written as [23]

$$\begin{aligned}
E(z,T) &= [E_t(T) - E_b(T)]\left(\frac{2z+h}{2h}\right)^N + E_b(T) \\
\alpha(z,T) &= [\alpha_t(T) - \alpha_b(T)]\left(\frac{2z+h}{2h}\right)^N + \alpha_b(T) \\
\nu(z,T) &= [\nu_t(T) - \nu_b(T)]\left(\frac{2z+h}{2h}\right)^N + \nu_b(T) \\
\rho(z) &= (\rho_t - \rho_b)\left(\frac{2z+h}{2h}\right)^N + \rho_b \\
\lambda(z) &= (\lambda_t - \lambda_b)\left(\frac{2z+h}{2h}\right)^N + \lambda_b
\end{aligned} \tag{5}$$

where the subscript t and b indicate the top surface and bottom surface of the plate, respectively.

We assume that the temperature variation occurs in the thickness direction only and one-dimensional temperature field is constant in the x–y plane of the plate. In such a case, the temperature distribution along the thickness can be obtained by solving a steady-state heat transfer equation

$$-\frac{d}{dz}\left[\lambda(z)\frac{dT}{dz}\right] = 0 \tag{6}$$

Considering the boundary conditions

$$z = h/2 \text{ at } T = T_t;\ z = -h/2 \text{ at } T = T_b \tag{7}$$

the solution of this equation, by means of polynomial series, is [24]

$$T(z) = T_b + (T_t - T_b)\eta(z) \tag{8}$$

and

$$\begin{aligned}
\eta(z) = \frac{1}{\vartheta}\Bigg[&\left(\frac{2z+h}{2h}\right) - \frac{\lambda_{tb}}{(N+1)\lambda_b}\left(\frac{2z+h}{2h}\right)^{N+1} + \frac{\lambda_{tb}^2}{(2N+1)\lambda_b^2}\left(\frac{2z+h}{2h}\right)^{2N+1} \\
&- \frac{\lambda_{tb}^3}{(3N+1)\lambda_b^3}\left(\frac{2z+h}{2h}\right)^{3N+1} + \frac{\lambda_{tb}^4}{(4N+1)\lambda_b^4}\left(\frac{2z+h}{2h}\right)^{4N+1} \\
&- \frac{\lambda_{tb}^5}{(5N+1)\lambda_b^5}\left(\frac{2z+h}{2h}\right)^{5N+1}\Bigg] \\
\vartheta = 1 &- \frac{\lambda_{tb}}{(N+1)\lambda_b} + \frac{\lambda_{tb}^2}{(2N+1)\lambda_b^2} - \frac{\lambda_{tb}^3}{(3N+1)\lambda_b^3} + \frac{\lambda_{tb}^4}{(4N+1)\lambda_b^4} \\
&- \frac{\lambda_{tb}^5}{(5N+1)\lambda_b^5}
\end{aligned}$$

where $\lambda_{tb} = \lambda_t - \lambda_b$.

From Eqs. (5) and (7), it can be seen that now E_t, α_t, ν_t, E_b, ν_b and α_b are all functions of temperature and position.