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Theoretical and numerical calculation of the acoustic radiation force acting on a circular rigid cylinder near a flat wall in a standing wave excitation in an ideal fluid

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ABSTRACT

The acoustic radiation force acting on a cylinder near a flat wall in a standing wave is calculated by analytical methods and numerical simulations. An exact theoretical solution is presented as well as an approximate solution. The approximate solution is in algebraic form and quite easy to compute. The numerical simulation is based on FVM (Finite Volume Method) on unstructured triangular meshes. The exact theoretical, approximate and numerical solutions are compared with each other and good agreements are obtained. Furthermore, the effects of the flat wall are investigated in detail by the three methods.

1. Introduction

It is well known that when an acoustic wave is absorbed or reflected by an object in a medium a small force, i.e. the acoustic radiation force, is exerted on the object. This type of force, like the phenomenon of acoustic streaming, is a non-linear effect of a sound field and arises from the momentum transfer between the sound field and the suspended particle. The acoustic radiation force can be applied to many practical fields such as acoustic sensors, ultrasonic levitation and contactless particle manipulation [1–3] which has become a hot research topic in ultrasonic devices and micromachined systems.

Undoubtedly, it is necessary to accurately predict the total forces on suspended particles including the primary force and the secondary force to design ultrasonic devices, e.g. ultrasonic particle manipulators. The wave length is usually much larger than the size of particles in practical applications. Therefore, Gor'kov's theory [4] is adopted to obtain the force potential field, from which the forces on particles can be obtained. Oberti et al. [3] demonstrated an interesting approach to determine the two-dimensional force potential patterns under two standing wave beams by Gor'kov's theory. He has shown that this method coincided with the experiments well. However, this method is not accurate near chamber walls, since the Gor'kov's theory does not include the effects of chamber boundaries. In some recent devices [5,6], researchers generated quarter-wavelength standing waves in a chamber, and positioned the particles near the chamber wall. The interactions between particles and chamber walls become very important in these cases, e.g. in connection with stiction. A strong need is therefore arising to predict the acoustic forces accurately near chamber walls in a standing wave.

The calculation of the acoustic radiation force on a particle arising from a sound field has attracted attention for a long time. King [7] first calculated the acoustic radiation pressure on a rigid sphere in an inviscid fluid exerted by planar traveling and standing waves. Then, Yosioka and Kawasima [8] extended King's theory to compressible spheres, and Hasegawa and Yosioka [9] to elastic particles. Gor'kov [4] proposed another more general and simple formula which can be used in practical applications conveniently. Recently, Mitri and Fellah [10] deduced new expressions for the radiation force acting on a sphere by the far-field scattering field. Although this approach has a simpler mathematical form, it is difficult to be extended to the cases with chamber walls and fluid viscosity, where the traditional approaches based on the near-field scattering field is necessary.

There also have been a few publications for the acoustic radiation forces on cylinders. Awatani [11] proposed the first calculation of the acoustic radiation forces acting on a rigid cylinder and Hasegawa et al. [12] for an elastic cylinder. Their calculations are all performed with an inviscid fluid and a planar traveling wave. Wu and Du [13] reported an analytical solution for a rigid cylinder in an inviscid fluid subjected to a standing wave field. They got a good agreement within 20% errors compared with their experiments. Haydock [14] followed King's original approach and formulated the radiation force on a rigid cylinder in an ideal fluid due to a standing wave. Haydock claimed that the solution was easy to be

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evaluated by using common numerical software packages such as MATLAB. Wei et al. [15] obtained the analytical formulations for compressible cylinders in a non-viscous fluid in a standing wave field by using the far-field acoustic scattering solution. Mitri and Chen [16] extended the far-field method to gain solutions for elastic and viscoelastic cylinders.

In the above papers, the force acting on one particle, namely the primary force, was considered. If two objects are close together in an acoustic field in a fluid, there exists another kind of force called the secondary force in addition to the primary force. The secondary force arises from the scattered acoustic field from the other particle. The research on the radiative interaction of particles was started by Bjerknes [17], who first derived an analytical expression for a time-averaged interaction force of two pulsating bubbles. Crum [18] carried out experiments and verified the Bjerknes' theory. Doinikov and Zavtrak [19] derived a formula for the interaction forces between two bubbles in an incompressible fluid without viscosity. Then, the compressibility of the host fluid was considered [20], which corresponds to eliminating the restriction that the distance between the two bubbles is much smaller than the wavelength. The mutual acoustic force between two bubbles in an incompressible and viscous fluid is investigated under non-slip [21] and slip [22] bubble boundary conditions, respectively. All of Doinikov's papers about bubbles assumed that the radii of the two bubbles are both much smaller than the wavelength and the center line of the two bubbles is parallel to the direction of the incident wave. Weiser and Apfel [23] calculated the mutual forces between two rigid and compressible spherical particles. Zheng and Apfel [24] formulated the forces between two spheres in an arbitrary plane wave field. The last two papers are both in ideal fluids.

Numerical work has appeared in recent years and is not as rich as the theoretical one, despite of its importance for more complex situations, like nonspherical particles. Townsend et al. [25] used CFD (Computational Fluid Dynamics) software and Gor'kov's theory to model the particle paths in a fluid in a ultrasonic standing wave. Their simulations did not really investigate the time-averaged forces on particles in detail. Neild et al. [26] modeled the acoustic pressure field in a real microparticle manipulator by FEM (Finite Element Method) simulation. However, they only computed the locations of the trapped particles instead of the real forces on particles. By using the LB (Lattice Boltzmann) method for solving the Navier–Stokes equations in the host medium, Cosgrove et al. [27] simulated the particle motion in an ultrasound field and compared the results with Wu and Du's [13] theoretical predictions. Haydock [28] also adopted the LB method to calculate the time-averaged forces on a cylinder in a standing wave and compared them with his analytical solutions [14]. However, it should be noted that their simulations both show significant deviations from the theoretical predictions. We proposed a numerical scheme based on the FVM (Finite Volume Method) and the PML (Perfect Matched Layer) algorithm in Ref. [29] for calculating the acoustic radiation force on a cylinder in a standing wave in an infinite region. The results agreed with the theoretical predictions very well.

In summary, there does not exist any detailed discussion on the interaction between a particle and a rigid flat wall in a standing wave. However, the effects of the chamber walls are very important in order to fully describe the force field within a practical particle manipulator. In this paper, we shall address this lack of theory and simulation in order to aid the design of ultrasonic devices. We first present an exact analytical solution for calculating the time-averaged forces acting on a rigid cylinder near a rigid flat wall in a standing wave in an inviscid fluid, and then an approximate solution are proposed in algebraic form under the assumptions that the particle size and the distance from the wall are both much smaller that the wave length. In addition, numerical simulations are carried out to verify the exact and approximate solutions. The results of the three methods are compared with each other and good agreement is achieved. The effects of the flat wall are investigated in detail by the analytical solutions.

2. Theory

A fixed infinite rigid cylinder of radius a is located at a distance d away from a planar rigid wall (Fig. 1) immersed in an infinite ideal fluid. A plane traveling wave is incident obliquely with the wave length λ and the incident angle α to the rigid wall (Fig. 1). If the incident angle $-\pi < \alpha < 0$, the incident traveling wave is reflected on the rigid wall and the incident and the reflected waves will interfere and produce a standing wave. In the cases of $\alpha = -\pi$ and $\alpha = 0$, another incident traveling wave is introduced in the opposite propagation direction of the original incident wave to generate a standing wave in x direction. We intend to calculate the total time-averaged force acting on the cylinder in these situations of incident wave. The method of images is employed by introduce an image cylinder and an image incident wave mirrored at the flat wall in order to satisfy the boundary conditions at the wall. As demonstrated in Fig. 1, two cylindrical coordinates systems (r_1, θ_1, z_1) and (r_2, θ_2, z_2) are established at the centers of the original cylinder (cylinder 1) O_1 and the image cylinder (cylinder 2) O_2, respectively. In the geometrical configuration of the image method in Fig. 1, the reflecting wave of the incident wave is substituted by the image incident wave and the reflecting waves of the scattering waves from the original cylinder is replaced by the scattering from the image cylinder.

The problem is formulated by the standard methods of theoretical acoustics using a velocity potential as [30]

$$\begin{aligned} &\mathbf{u} = \nabla\varphi \\ &p = -\rho_0\dot{\varphi} \\ &\nabla^2\varphi + k^2\varphi = 0, \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the velocity vector of the fluid, φ is the velocity potential, p is the fluid pressure, ρ_0 is the density of the undisturbed state, $k = \omega/c$ is the wave number, and c is the speed of sound in the fluid. Here, the harmonic time factor $e^{-i\omega t}$ is suppressed for simplicity.

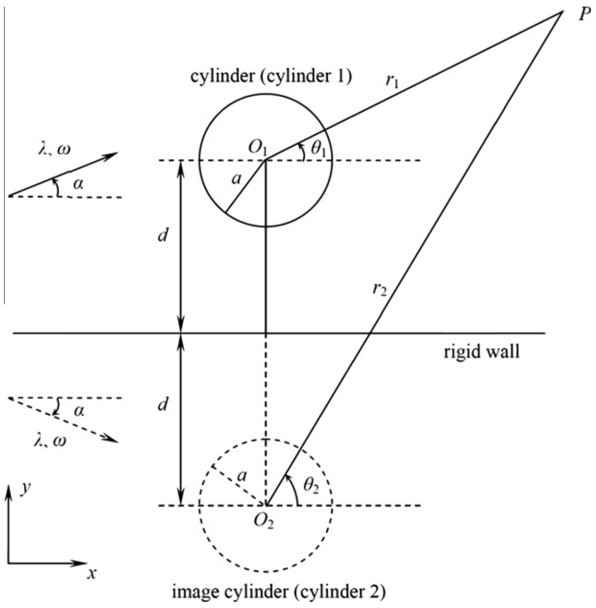

Fig. 1. Geometrical configuration for theoretical solution.