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Estimation of complex arterial elastic modulus from ring resonance excited by ultrasound radiation force

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Abstract

Pulse wave velocity (PWV) is widely used for estimating the stiffness of an artery. PWV is an average measurement of artery stiffness between two measuring sites. From measured PWV, the diameter and thickness are needed to calculate the elastic modulus of the artery. In this paper a new method of using ring resonant mode for estimation of arterial elastic modulus is proposed. To generate the ring resonance, a localized radiation force of ultrasound is remotely and non-invasively applied at the artery. The vibration response of the artery is measured by optical techniques. Three ring resonant modes are identified for estimation of the elastic modulus. The viscoelasticity and the complex modulus of the artery can be obtained. Experiments were carried out on a porcine artery embedded in gelatin. The estimation only requires the diameter of the artery, but does not need the thickness of the artery which is difficult to measure with accuracy and precision.

Keywords: Artery; Pulse wave velocity; Elastic modulus; Ring resonance; Ultrasound

1. Introduction

The mechanical properties of the arterial vessels are very important because they influence arterial physiology and the development and progression of arterial diseases via effects on blood flow and arterial mass transport [1]. It has long been recognized that a high percentage all cardiovascular disease is associated with a hardening of the arteries or arteriosclerosis [2]. Increased stiffness of the arteries has recently gained acceptance as a fundamental risk factor for cardiovascular and many other diseases [3].

Pulse wave velocity (PWV) is widely used for estimating the stiffness of an artery. PWV is measured using the ‘‘foot-to-foot'' method. However, the ‘‘foot'' of the pressure wave is affected by the reflected waves. PWV is an average measurement of artery stiffness between the two measuring sites and, therefore, does not identify local stiffness. In addition to PWV, the diameter and thickness of the artery are needed to calculate the elastic modulus of the artery. We have recently developed several novel methods for non-invasively estimation of elastic properties of arteries [4,5].

In this paper we propose to excite the so-called ‘‘ring resonance'' in the artery wall by localized radiation force of ultrasound. By measuring the ‘‘ring resonant frequency'', the Young's modulus of the artery is determined. One major advantage is that estimation of Young's modulus is independent of artery wall thickness. The well-known pulse wave velocity technique however, requires both the diameter and wall thickness for estimating Young's modulus. Measurement of artery wall thickness is much more difficult than measurement of artery diameter.

2. Theory

A new theory for studying the ring resonant modes in an artery is presented in which the artery is considered as a three-dimensional elastic cylindrical tube (Fig. 1(a)). The equations of motion for a homogeneous, isotropic, and

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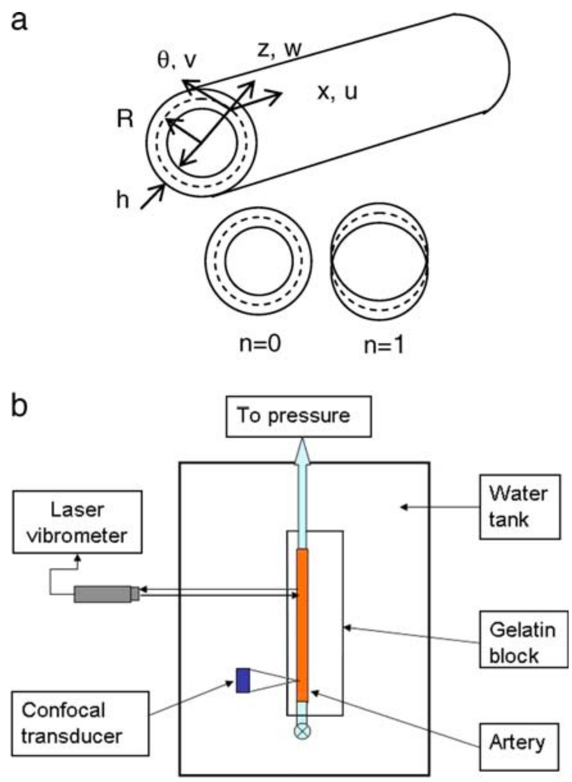

Fig. 1. (a) Three dimensional coordinate system, mode $n=0$ and mode $n=1$. (b) Schema of the experimental system for measuring the ring resonance frequency in an artery with laser.

linear elastic cylindrical tube can be written by Love's theory as [6]

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{1}$$

where u, v, w are, respectively, the displacements of the tube in the x, θ, z directions, L_{ij} $(i,j = 1,2,3)$ are differential operators [6].

The equation of the ring resonant frequency can be written as

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} U_n \\ V_n \\ W_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{2}$$

where $C_{11} = \Omega^2 - \frac{(1-v)n^2}{2}, C_{12} = 0, C_{13} = 0, C_{21} = C_{12}$, $C_{22} = \Omega^2 - (1+\beta)n^2$, $C_{23} = -n - \beta n^3$, $C_{31} = C_{13}$, $C_{32} = C_{23}$, $C_{33} = \Omega^2 - \beta n^4 - 1$, U_n, V_n and W_n are respectively the vibration amplitudes, $\beta = h^2/(12R^2)$, h is the thickness, R the median radius, Ω is the non-dimensional frequency parameter, $\Omega = \omega R/c_p$, and $c_p = \sqrt{E/[\rho(1-v^2)]}$, E the Young's modulus, v the Poisson's ratio, ρ the mass density, and n is the circumferential mode and ω the circular frequency.

Resonant frequencies can be solved with Eq. (2) for different circumferential modes n. Three resonant frequencies are found at $\Omega = 0.5$, 1, $\sqrt{2}$, respectively. The frequency equation can be written from $\Omega_{0.5} = 0.5$ as

$$f_{0.5} = \frac{1}{2\pi R}\sqrt{\frac{E}{2\rho(1+v)}}. \tag{3}$$

The frequency equation can be written from $\Omega_1 = 1$ as

$$f_1 = \frac{1}{2\pi R}\sqrt{\frac{E}{\rho(1-v^2)}}. \tag{4}$$

The frequency equation can be written from $\Omega_{\sqrt{2}} = \sqrt{2}$ as

$$f_{\sqrt{2}} = \frac{\sqrt{2}}{2\pi R}\sqrt{\frac{E}{\rho(1-v^2)}}. \tag{5}$$

3. Experiment

The radiation force of ultrasound is used to generate a force remotely in an artery. For this purpose two intersecting CW focused ultrasound beams of different frequencies are used to generate a localized radiation force [4,5]. This force is modulated at the difference frequency of the two ultrasound beams, thus, a low frequency localized force is produced which excites mechanical vibration modes of the artery.

A schema of the experimental setup is shown in Fig. 1(b). A fresh artery is embedded in a tissue–mimicking gelatin mixture. The artery phantom is placed in a water tank. The artery can be pressurized by internal saline through a connecting rubber tube. A 3 MHz confocal transducer of 10 cm focal length produces the radiation force remotely on the artery. The focal size of the ultrasound beam is about 0.7 mm in diameter which generates almost a point force on the artery. The vibration of the artery is measured noncontact with a laser vibrometer.

4. Results and discussion

4.1. Ring resonant modes

A fresh porcine artery had a length of 10 cm and an outer diameter of 6 mm. The artery was sealed in a gelatin phantom box and pressurized internally with saline to 70 mm Hg. A harmonic localized radiation force of ultrasound was non-invasively applied at the artery. When the excitation force was swept over a frequency range from 100 to 2000 Hz, the frequency spectrum of the vibration was measured by laser from which the resonant frequencies were determined. Fig. 2 shows experimental results of the frequency spectrum of the artery. The frequency resolution is 1.5 Hz. The three ring resonant frequencies are, respectively, $f_{0.5} = 356$ Hz, $f_1 = 718$ Hz, and $f_{\sqrt{2}} = 968$ Hz. The identification of the modes is based on experimental modal analysis technique and our theoretical model. The sharp peak at 900 Hz is a previously identified electronic system noise source in the laser vibrometer.