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Modelling nonlinearity in piezoceramic transducers: From equations to nonlinear equivalent circuits

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ABSTRACT

Quadratic nonlinear equations of a piezoelectric element under the assumptions of 1D vibration and weak nonlinearity are derived by the perturbation theory. It is shown that the nonlinear response can be represented by controlled sources that are added to the classical hexapole used to model piezoelectric ultrasonic transducers. As a consequence, equivalent electrical circuits can be used to predict the nonlinear response of a transducer taking into account the acoustic loads on the rear and front faces. A generalisation of nonlinear equivalent electrical circuits to cases including passive layers and propagation media is then proposed. Experimental results, in terms of second harmonic generation, on a coupled resonator are compared to theoretical calculations from the proposed model.

Contents

1. Introduction

The study of nonlinear properties of materials has become an important topic within the scientific community. The reason is that nonlinear parameters can provide additional information of interest on the status of these materials, such as water content [1] or state of fatigue [2], [3]. In this last field, authors have used the fact that the nonlinear parameter, associated to harmonic generation, is a sensitive indicator of the state of damage of a material. One of the issues of Non-Destructive Evaluation (NDE) is then to characterize the life time of the material before the onset of significant defects.

However, as nonlinear effects in solids can be very low compared to linear phenomena, the measurements can easily be disturbed by the existence of external sources of nonlinearity. Several NDE measurement methods are based on the spectral modification of an acoustic wave propagating through a sample under test. External nonlinearities can come from the electrical set-up and/or from the transducers used for emitting or receiving the ultrasonic waves. If the effect of the electronics can be reduced by the use of an adapted filtering stage, the transducers can be a recurrent problem. Thus, authors [2,4] have shown that, in the range of intensities (strain and stress in the transducer respectively of the order of 10^{-5} and 10^{6} MPa) commonly used for NDE applica-

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tions, these nonlinearities are competing with those produced by the propagation medium and can severely affect the accuracy of nonlinear measurements.

The aim of this work is the modelling of a nonlinear instrumentation chain in the context of a weak nonlinearity. In this study, nonlinear equivalent electrical circuits are developed from a classical model [5] in order to obtain a tool allowing nonlinear acoustic, electric and piezoelectric elements to be taken into account. Due to its anisotropy, the global characterization of the nonlinear properties of the piezoceramic constituting the active element of the transducer can become a very complex problem involving a large number of nonlinear parameters and high order constants [6]. However, in many applications, the ultrasonic transducer including, in addition to the piezoelectric element, a backing and a matching layer operates in a onedimensional mode, generally the thickness-mode or the length-extensional mode. In these cases, previous experimental results [7–9], have shown that the nonlinear behavior of the piezoelectric element can be related to one or two nonlinear parameters expressing the electromechanical and the mechanical origins of the intrinsic nonlinearity in the first-order approximation. In this study, the length-extensional mode is presented in order for realistic nonlinear parameters extracted from experimental results concerning PZT ceramics [9] to be introduced.

2. General statements

Due to the weak nonlinearity assumption, the constitutive equations of the piezoelectric element will be developed up to quadratic terms only. Besides, as it has also been mentioned, most of the transducers operate in a onedimensional mode. The purpose of this section is to expose the usefull constitutive equations governing such onedimensional models. For example, in the case of the length-extensional geometry, the usefull relations, in the first nonlinear approximation, between the displacement u along the z-axis of a piezoelectric rod, the first Piola–Kirchhoff stress T, the electrical field E and the electrical displacement D are formally [7]:

$$\begin{aligned} \frac{\partial u}{\partial z} &= sT + gD + \frac{s'}{2}T^2 - \alpha DT - \frac{\gamma}{2}D^2 \\ E &= \beta D - gT + \frac{\alpha}{2}T^2 + \frac{\beta'}{2}D^2 + \gamma DT \end{aligned} \tag{1}$$

where s, g and β are respectively the compliance, piezoelectric and inverse of permittivity second-order constants. The third-order constants s' and β' express respectively second-order elastic and dielectric effects whereas α and γ are related to an electromechanical nonlinearity. According to [10], second-order dielectric effects can be ignored. Besides, as D does not depend on space in the quasi-static approximation of Maxwell's law:

$$\frac{\partial D}{\partial z} = 0 \tag{2}$$

the constant γ plays no role in the dynamic law which provides the following equation between the local acceleration and the first Piola–Kirchhoff stress – ρ being the reference mass density:

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial z}T \tag{3}$$

In addition, previous experimental results have confirmed [9] the predominance of the mechanical nonlinearity associated to the constant s' and of the electromechanical nonlinearity associated to the constant α in the length-extensional mode. The constant γ can then also be ignored in a first approach. Thus, the usefull constitutive relationships in this geometry can be reduced to:

$$\begin{aligned} \frac{\partial u}{\partial z} &= sT + gD + \frac{s'}{2}T^2 - \alpha DT \\ E &= \beta D - gT + \frac{\alpha}{2}T^2 \end{aligned} \tag{4}$$

The hypothesis of a weak nonlinearity allows nonlinearity to be considered as a perturbation of linear fields. For each quantity u, E, T and D involved in (Eq. (4)), the general solution of the nonlinear problem is searched as respective sums:

$$u = \sum_{i=0}^{\infty} u_i, \quad E = \sum_{i=0}^{\infty} E_i, \quad T = \sum_{i=0}^{\infty} T_i \quad \text{and} \quad D = \sum_{i=0}^{\infty} D_i$$

according to the method of the little parameter [11]. In the previous expansion, the subscript $i = 0$ represents the solution of the linear problem and the subscript $i \geqslant 1$ represents the solution calculated at the $i \geqslant 1$ order of nonlinearity by successive approximations [11]. Note that this method supposes that each quantity at the order $i + 1$ is small compared to the respective one at the order i, which implies, on one hand, that linear amplitudes are small enough and, on the other hand, an absence of internal resonance. In this work, the solution of the nonlinear problem will be approached at the first-order of nonlinearity only. The linear problem, associated to the zero-order quantities, will be first solved (Section 3). This solution will then be used to solve the problem associated to the first-order quantities (Section 4).

3. Linear modelling: impedance matrix

In this section, the geometry and the physical quantities being defined, the classical solution [12] of the linear problem associated to the zero-order quantities is exposed in terms of matrix relations which will be used by following. The constitutive equations are those of (Eq. (4)) reduced to linear terms:

$$\begin{aligned} \frac{\partial u_0}{\partial z} &= sT_0 + gD_0 \\ E_0 &= \beta D_0 - gT_0 \end{aligned} \tag{5}$$

The combination of (Eq. (5)) with the dynamic law (Eq. (3)) and the Maxwell-law (Eq. (2)) leads to the homogeneous wave equation:

$$\rho\frac{\partial^2 u_0}{\partial t^2} - \frac{1}{s}\frac{\partial^2 u_0}{\partial z^2} = 0 \tag{6}$$

The piezoelectric rod is defined by its length $2a$ and a cross-section area $\Sigma = hd$ with $h, d \ll a$ (Fig. 1). It is supposed to be driven by a voltage applied at its ends $z = \pm a$.

The particle velocity $v_0(\pm a, t)$, the stress $T_0(\pm a, t)$ at each face, the electrical current I_0 – defined by $I_0(t) = \Sigma\frac{dD_0}{dt}$ – and the applied voltage $V_0(t)$ are physical quantities which are related to each other. The relations between these quantities can be formally described by an hexapole H_0 with two acoustical ports and an electrical port. In the case of a passive layer, similar relations between stress and particle velocities at each face leads to a simple quadrupolar representation Q_0 since there is no electrical port. Besides, the continuity of stress and particle velocity at each interface imposes connections between one quadrupole and the next as well as connections between the hexapole and each of the first quadrupoles located at its left or right.

In case of a sinusoidal electrical excitation at an angular frequency ω at the connections $z = \pm a$, $V_0(t) = V_0 exp(j\omega t)$, the particle velocity in the piezoelectric medium can be expressed as the sum of two waves respectively in $j(\omega t + kz)$ and $j(\omega t - kz)$:

$$v_0(z, t) = [A_+ e^{jkz} + A_- e^{-jkz}]e^{j\omega t} \tag{7}$$

where $k = \omega c$ is the wave number, $c = \sqrt{\frac{1}{\rho s}}$ being the longitudinal wave velocity in length-extensional mode. Introducing the acoustical impedance $Z = \rho c$ of the piezoelectric layer and the phase term