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Ultrasonic wave propagation in cementitious materials: A multiphase approach of a self-consistent multiple scattering model

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ABSTRACT

This paper examines ultrasonic wave propagation through strongly heterogeneous materials such as cementitious materials, and deals meanly with the formulation of a multiphase approach of a self-consistent multiple scattering model, the so-called dynamic generalized self-consistent model (DGSCM) proposed by Yang [J. Appl. Mech. 70(2003) 575–582]. This extended model can describe the influence of the size and volume fraction of aggregates on cementitious materials, as well as the interaction, contribution, and influence of entrapped air voids together with the aggregates on frequency-dependent parameters such as the phase velocity and the attenuation coefficient. To show the performance of this approach, theoretical predictions were compared with experimental ultrasonic measurements over a wide frequency range from several mortar specimens with different features in their microstructure properties and concentrations of aggregates up to 60%. The multiphase approaches of both the DGSCM and the Waterman–Truell model (WT) were also compared. The obtained results of the multiphase DGSCM were found to be significantly better than those obtained from the N-phase WT model for ultrasonic measurements from cementitious materials at high aggregate concentrations. The feasibility of material characterization using the multiphase approach of DGSCM was also discussed.

1. Introduction

Concrete is one of the most commonly composite materials. It is composed of a cement matrix that acts as a binder surrounding aggregates of different size and shape (fine and coarse aggregates). Cementitious materials such as mortar and concrete are strongly heterogeneous materials, the quality of which is normally determined by the compressive strength and the required homogeneity regarding the distribution of their phase constituents in the material microstructure. This quality depends on several factors, such as the type and proportion of the cement and the aggregates in the mix, the water-to-cement (w/c) ratio, and porosity, among others. Assessment of durability in cementitious materials mainly involves the environmental conditions and the aggressive agents to which they will be exposed, as well as the resulting microstructure of the mixed materials (mechanical and physical properties).

Ultrasonic non-destructive testing techniques have proven to be effective in evaluating the mechanical and physical properties of these kinds of materials, and their appropriate interpretations have led to progress in assessments of both quality and durability. It is still, however, necessary to have suitable models that allows for a more precise description of the ultrasonic waves as they travel through cementitious materials. This description must take into account the influence of the size and volume fraction of aggregates on the composites, as well as the air voids, because the occurrence of entrapped air voids in the manufacturing process of such materials is inevitable. Therefore, models for the study of multiphase materials are clearly needed.

To describe the ultrasonic mean field propagating through the material (i.e., mortar) the scheme of effective medium can be used. It can be seen as a dynamic homogenization process, as shown in Fig. 1, where effective parameters of the average wave field are related to effective mechanical and physical properties of the material. In this study, mortar is considered a three-phase material consisting of a solid phase (viscoelastic cement matrix), another phase consisting of elastic inclusions (fine aggregates) and a fluid phase that takes entrapped air voids (cavities) into account, as shown in Fig. 1.

Mortar behaves as a dispersive material in which the size of aggregates is assumed to be much greater than the characteristic capillary pore size, but smaller than or comparable with the ultrasonic wavelength. Thus, wave propagation through composite materials can be described by means of the effective longitudinal complex wave number $\langle k(\omega)\rangle$ that depends on the effective longitudinal phase velocity $\langle V_L(\omega)\rangle$ and the effective longitudinal

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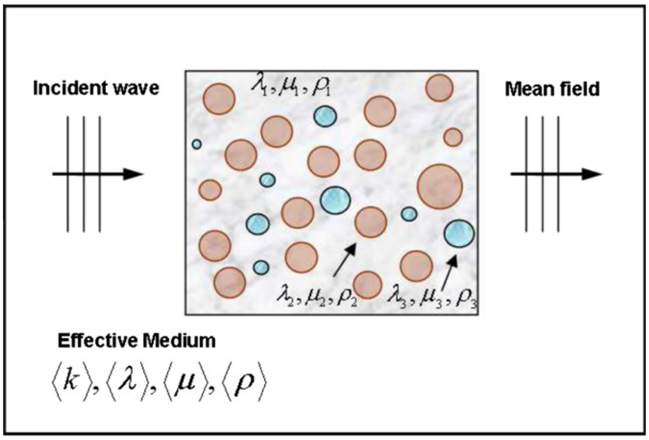

Fig. 1. Scheme of the effective medium.

attenuation $\langle\alpha_L(\omega)\rangle$. Such parameters are frequency-dependent as a consequence of wave scattering phenomena, and they can be related with $\langle k(\omega)\rangle$ as:

$$\langle k(\omega)\rangle = \frac{\omega}{\langle V_L(\omega)\rangle} + i\langle\alpha_L(\omega)\rangle \tag{1}$$

Furthermore, $\langle k(\omega)\rangle$ is related to the effective elastic constants such as Lamé modulus $\langle\lambda\rangle$ and shear modulus $\langle\mu\rangle$, as well as with the effective density $\langle\rho\rangle$:

$$\langle k(\omega)\rangle = \omega\sqrt{\frac{\langle\rho\rangle}{\langle\lambda\rangle + 2\langle\mu\rangle}} \tag{2}$$

Wave scattering phenomena involve both single and multiple scattering theories. Single scattering theory considers a low density of inclusions so that an incident wave is scattered by a single inclusion that is not affected by the presence of other inclusions. Multiple scattering theory deals with high concentrations of inclusions, and the interactions among them must be taken into account. Theoretical and experimental investigations based on these theories have been conducted by many authors to improve the understanding of wave propagation through cementitious materials. For example, Punurai et al. [1,2] estimated the size and volume fraction of entrapped and entrained air voids in cement pastes using ultrasonic attenuation modelled by a single scattering approach [3]. Aggelis et al. [4] conducted exhaustive theoretical and experimental studies for velocity and attenuation in fresh mortar using the well-known model formulated by Waterman and Truell (WT) [5]. Chaix et al. [6] employed the same model to evaluate microcracks in thermally damaged concrete. However, such models presented limitations at high concentrations of inclusions.

Several studies in the multiple scattering framework have been conducted to overcome such a problem, such as a self-consistent formulation of the WT proposed in [7], the self-consistent effective medium approximation formulated in [8], the iterative effective medium approximation (IEMA) [9], and the so-called dynamic generalized self-consistent model (DGSCM) [10,11]. The last model has shown its effectiveness to analyze a large number of different composite materials [12–17], but it has only been established for biphase materials.

This paper aimed to formulate a simple multiphase approach of the self-consistent multiple scattering model proposed by Yang in [11] for any type of spherical particulates (elastic, fluid or cavities). With this model, a theoretical study was conducted to describe the influence of the size and volume fraction of aggregates on cementitious materials, as well as the interaction, contribution, and influence of entrapped air voids together with the aggregates on frequency-dependent parameters such as the phase velocity and the attenuation coefficient. To show the performance of this approach, theoretical predictions were compared with experimental ultrasonic measurements over a wide frequency range from several mortar specimens with different features in their microstructure properties and concentrations of aggregates up to 60%. Furthermore, a brief discussion of feasibility of material characterization by means of this model is presented.

2. Multiphase approach of the self-consistent multiple scattering model

According to the multiple scattering model formulated by Waterman and Truell [5], the effective complex longitudinal wave number $\langle k(\omega)\rangle$ of the coherent waves in a composite including spherical inclusions, and generalized to different types of inclusions with different size distributions, is given by [18]:

$$\left(\frac{\langle k(\omega)\rangle}{k_1}\right)^2 = 1 + \frac{3}{k_1^2}\sum_j \frac{\phi_j}{a_j^3} f_j(0) + \frac{9}{4k_1^4}\sum_j \frac{\phi_j^2}{a_j^6}\left(f_j^2(0) - f_j^2(\pi)\right) \tag{3}$$

where k_1 is the longitudinal wave number of the matrix, ϕ_j and a_j are the volume fraction and radius, respectively, of each of the j different types of inclusions in the composite, and $f_j(0)$ and $f_j(\pi)$ are the complex forward and backward scattering amplitudes, respectively, for longitudinal waves due to the j-type of spherical inclusion embedded in an infinite matrix. In a wide frequency range, however, the WT model is assumed to be valid for a concentration of inclusions up to 40%. To overcome this limitation, the so-called dynamic generalized self-consistent model (DGSCM) was formulated [10,11]. This model assumes that each inclusion of radius a is surrounded by a matrix shell of inner radius a and outer radius b, which in turn is embedded in an effective medium with unknown properties. The volume fraction of inclusions is related to the radii as $\phi = a^3/b^3$. Therefore, this formulation consists of replacing the longitudinal and shear wave numbers in the matrix, k_1 and κ_1, by the effective longitudinal and shear wave numbers, $\langle k\rangle$ and $\langle\kappa\rangle$, respectively, from WT models for longitudinal and shear waves, yielding [11]:

$$1 = \left[1 + \frac{2\pi n_0}{\langle k\rangle^2} f(0)\right]^2 - \left[\frac{2\pi n_0}{\langle k\rangle^2} f(\pi)\right]^2 \tag{4}$$

$$1 = \left[1 + \frac{2\pi n_0}{\langle\kappa\rangle^2} g(0)\right]^2 - \left[\frac{2\pi n_0}{\langle\kappa\rangle^2} g(\pi)\right]^2 \tag{5}$$

where $g(0)$ and $g(\pi)$ are the forward and backward scattering amplitudes for shear waves. As these expressions are implicitly dependent on the unknown effective complex wave numbers, they are solved iteratively until sufficient convergence is reached. This model is based on combining the generalized self-consistent scheme formulated by Christensen and Lo [19] and the multiple scattering theory given by Waterman and Truell. However, Kim [12] emphasized that the iterative procedure by Yang has a physical interpretation concerning the forward scattering amplitude, which is proportional to the total extinguished energy due to the scatterers or inclusions. It therefore tends to vanish when the inclusions are only surrounded by the effective medium because the propagating medium is not the matrix, but the inclusions. Also Kim reported that the solution of this model can be reached when $f(0) = 0$. Either way, the method formulated by Yang, even though it considers both incident longitudinal and shear waves for cylindrical [10] and spherical inclusions [11], has only been established for two-phase materials. Therefore, a multiphase approach of the DGSCM is formulated in this study.