



Interaction of a high-order Bessel beam with a submerged spherical ultrasound contrast agent shell – Scattering theory

F.G. Mitri *

Mayo Clinic, College of Medicine, Department of Physiology and Biomedical Engineering, Ultrasound Research Laboratory, 200 First Street SW, Rochester, MN 55905, USA

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ABSTRACT

Background and objective: Acoustic scattering properties of ultrasound contrast agents are useful in extending existing or developing new techniques for biomedical imaging applications. A useful first step in this direction is to investigate the acoustic scattering of a new class of acoustic beams, known as helicoidal high-order Bessel beams, to improve the understanding of their scattering characteristics by an ultrasound contrast agent, which at present is very limited.

Method: The transverse acoustic scattering of a commercially available albuminoid ultrasound contrast agent shell filled with air or a denser gas such as perfluoropropane and placed in a helicoidal Bessel beam of any order is examined numerically. The shell is assumed to possess an outer radius $a = 3.5$ microns and a thickness of ~ 105 nm. Moduli of the total and resonance transverse acoustic scattering form functions are numerically evaluated in the bandwidth $0 < ka \leq 3$, which corresponds to a frequency bandwidth of 0–205 MHz that covers a wide range of applications for imaging with contrast agents. Particular attention is paid to the shell's material, the content of its interior hollow region and the fluid surrounding its exterior. The contrast agent shell is assumed to be immersed in an ideal compressible fluid so the viscous corrections are not considered. Analytical equations are derived and numerical calculations of the total and resonance form functions are performed with particular emphasis on the effect of varying the half-cone angle, the order of the helicoidal Bessel beam as well as the fluid that fills the interior hollow space.

Results and conclusion: It is shown that shell wave resonance modes can be excited on an encapsulated micro-bubble. The forward and backscattering vanish for a helicoidal high-order Bessel beam. Additionally, the fluid filling the inner core affects the shell's response significantly. Moreover, there is no monopole contribution to the axial scattering of a helicoidal Bessel beam of order $m \geq 1$ so that the dynamics of contrast agents would be significantly altered. The main finding of the present theory is the suppression or enhancement for a particular resonance that may be used to advantage in imaging with ultrasound contrast agents for clinical applications.

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1. Introduction

The study of sound scattering by contrast agents constitutes a noteworthy field of research in a wide number of diagnostic and clinical applications [1–9]. Such agents are injected intravenously and used to enhance the scattering signals from regions in which the blood circulates. These agents enable improved imaging capabilities to differentiate the blood from tissue. The geometrical shapes of such agents take the form of encapsulated spherical shells with a diameter ranging from 1 to 10 microns, with the inte-

rior space filled with either air or molecular gases of higher density (i.e. decafluorobutane or other) [10]. An elastic or viscoelastic shell of 10 to ~ 250 nm thickness encloses the outer surface. The shell's layer prevents contrast agent's dissolution and disintegration during capillary circulation.

Extensive efforts on predicting the ultrasonic signature of such agents have been concentrated on studying the acoustic scattering of progressive plane waves [11–14]. In most of those studies, the incident wave was assumed to be an infinite plane wave [15–18], so that the scattering does not depend on the beam shape. However, when acoustic illumination is in the form of a localized beam, such as in the field of a focused transducer [19,20], excitation of the resonances depends on the beam parameters. This effect has also been investigated using a zero-order Bessel beam incident upon spheres [21–23]. It is shown in those studies that the effect of varying the beam half-cone angle modifies the coupling to specific

* Fax: +1 507 266 0361.

E-mail addresses: mitri@ieee.org, mitri.farid@mayo.edu

resonances in the response of each target type. Unlike the case of plane wave illumination where selective excitation or extinction of a particular resonance can be hardly achieved, the acoustic scattering and radiation force of high-order Bessel beams are reduced or increased based on the target's mechanical properties and beam shape.

Bessel beams [24] denote a class of exact solutions of the homogeneous (source-free) Helmholtz equation [25]. However, in real situations when the beam generation is taken into account, the diffraction effects cannot be overcome and only approximations to the nondiffracting beams can be obtained. Such beams are usually called pseudo-nondiffracting or limited-diffracting beams. This character of limited diffraction beams is distinct from the conventional diffracting beams that remain diffracting no matter how big the aperture and how large the amount of energy used to produce them. Since such beams focus over a large depth of field, they have attracted various applications in medical imaging and tissue characterization [26,27]. On the other hand, significant research has been paid to the applications of high-order helicoidal Bessel optical beams [28–30]. Unlike the zero-order Bessel beam, helicoidal high-order Bessel beams (HOBB) have an axial phase singularity at the transverse origin where the intensity vanishes and about which the phase varies continuously by $\pm 2\pi m$, where $m = 1, 2, 3, \dots$. Due to this property, the HOBBs are labelled as “hollow” beams. Moreover, such beams with azimuthal dependence (i.e. $e^{\pm im\phi}$) carry an intrinsic orbital angular momentum similar to the one caused by helicoidal waves [31].

Most of the theoretical investigations of HOBBs involve the interaction of waves with spherical particles [21,22,32–42]. Although a theoretical study has been recently developed to study the acoustic scattering of a HOBB [34], it was however limited to the interaction of a Bessel acoustic beam of order m with an elastic sphere. In that study, the transverse acoustic scattering field was investigated for specific dimensionless frequencies and beam half-cone angles. For HOBBs, the acoustic scattering vanishes in the backward ($\theta = \pi$) and forward ($\theta = 0$) directions along the beam axis [34]. Moreover it was possible to suppress the excitation of certain resonances of an elastic sphere by appropriate selection of the beam parameters [23,32,34,35].

Motivated by investigating the properties of HOBBs for possible biomedical and biophysical applications, analytical and numerical analyses are undertaken here to extend the previous study [34] to the case of a spherical shell (with hollow) of a particular thickness. Theoretical and numerical predictions for the acoustic scattering of high-order Bessel acoustic beams by spherical shells may be an asset in contrast agent design because they adequately model the acoustic response of shells. Simulations are more practical to build in a shorter amount of time and predictions may be effective in showing important factors to optimize the production of contrast agents for experimental and clinical testing.

The focus of this investigation is to study the characteristics of scattering from a contrast agent shell when subjected to helicoidal Bessel acoustic beams of any order. The examples chosen herein are relevant to medical ultrasound applications. The contrast agent shell must remain relatively thin and flexible enough to allow for transport through the capillary blood vessels. In this model, the shell is placed along the axis of a HOBB and immersed in an ideal (non-viscous) fluid. Examples for the transverse acoustic scattering are studied versus the beam parameters (order of the beam m and half-cone angle β) and the gas that fills the inner core of the shell. General properties of the scattering theory by a contrast agent shell are discussed in Section 2 and the example of an albuminoidal spherical shell in water is discussed in Section 3. The findings are summarized in Section 4.

2. Acoustic scattering of a high-order Bessel beam from a spherical shell

The center of a spherical shell of outer radius a , inner radius b , is located at the z axis of a coordinate system. The exterior and interior fluid densities are ρ_{ext} and ρ_{int} , respectively, and the shell's density is denoted by ρ_s . The spherical shell is immersed in an ideal compressible fluid, so that viscous and thermal effects can be neglected.

The acoustic pressure produced by an ideal helicoidal high-order Bessel beam (HOBB) can be described by [34]

$$p_B^{(\text{inc})} = p_0 e^{i(k_z z - \omega t)} J_m(k_r R) e^{\pm im\phi}, \quad (1)$$

where p_0 is the pressure amplitude, $J_m(\cdot)$ is an m th-order Bessel function, $k_z = k \cos \beta$ and $k_r = k \sin \beta$ are the axial and radial wave numbers, $k = \sqrt{k_z^2 + k_r^2} = \omega/c = 2\pi/\lambda$, is defined as the wave number of the incident beam in the exterior fluid, ω is the angular frequency, c is the speed of sound in the fluid medium, λ being the wavelength of the acoustic radiation making up the HOBB, β is the half-cone angle formed by the wave number k relative the axis of wave propagation, and $R = r \sin \theta$, where r , ϕ , and z are the radial, azimuthal, and axial components respectively.

In a system of spherical coordinates, the incident pressure represented by a cylindrical wave function given by Eq. (1) can be expressed in spherical coordinates as [34,38]

$$p_B^{(\text{inc})} = p_0 e^{-i\omega t} \sum_{n=m}^{\infty} \frac{(n-m)!}{(n+m)!} (2n+1) i^{n-m} j_n(kr) P_n^m(\cos \theta) P_n^m(\cos \beta) e^{\pm im\phi}. \quad (2)$$

Eq. (2) describes the pressure of a HOBB incident upon a spherical shell, whose center is located at a distance r from an observation point, $j_n(\cdot)$ is the spherical Bessel function of order n , $P_n^{(m)}(\cdot)$ are the associated Legendre functions of the first kind of degree n and order m , and θ is the scattering angle relative to the beam axis of wave propagation z .

The radial displacement associated with the incident pressure is expressed by [43]

$$u_r = \left(\frac{1}{\rho_{\text{ext}} \omega^2} \right) \frac{\partial p_B^{(\text{inc})}}{\partial r}. \quad (3)$$

The material displacement due to the interaction of the acoustic beam with the shell is represented in terms of scalar and vector potentials as follows;

$$\mathbf{u}_{\text{int}} = -\nabla \Phi_{\text{int}} + \nabla \times \Psi_{\text{int}}, \quad (4)$$

where Φ_{int} and Ψ_{int} ($\Psi_r = 0$, $\Psi_\theta = 0$, $\Psi_\phi = \Psi_{\text{int}}$) are expressed in spherical coordinates by

$$\Phi_{\text{int}} = p_0 e^{-i\omega t} \sum_{n=m}^{\infty} \left\{ \frac{(n-m)!}{(n+m)!} (2n+1) i^{n-m} [A_n j_n(k_l r) + B_n y_n(k_l r)] \times P_n^m(\cos \theta) P_n^m(\cos \beta) e^{\pm im\phi} \right\}, \quad (5)$$

and

Table 1

Material parameters used in the numerical calculations.

Material	Mass density (kg/m ³)	Compressional velocity c_L (m/s)	Shear velocity c_S (m/s)
Albuminoidal protein	1100	7458	284
Perfluoropropane (gas)	90.244	92.062	N/A
C ₃ F ₈ at 26.85 °C (Ref. [55])			
Air	1.23	340	N/A
Water	1000	1500	N/A

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