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Modeling of flexible waveguides for ultrasonic vibrations transmission: Longitudinal and flexural vibrations of non-deformed waveguide

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ABSTRACT

The article presents the mathematical model allowing to investigate longitudinal and flexural vibrations of stepped flexible waveguides with transitional section without regard to various vibration modes interaction. The model uses original numerical-analytic calculations based on analytical solutions of the equation of waveguide steps vibrations and their continuous matching with numerical solution of the equation of transitional section vibrations. The proposed model can be considered as an initial approximation to the solution of the problem of flexible waveguides design, which makes it possible to determine and validate effective methods of its addressing. Resonant curves of longitudinal and flexural vibrations of two-step waveguide are traced for the given vibration frequency. Step lengths values providing simultaneous resonance of longitudinal and flexural vibrations for the given frequency are determined. Validity of the proposed model is proved by the results of finite elements method (FEM) modeling using ANSYS® software. Application of Timoshenko's model instead of Euler–Bernoulli's model for description of flexural vibrations enabled reduction of relative deviation of resonant frequencies calculated using ANSYS® from the value specified during resonant curves tracing down to negligible value (0.17%).

1. Introduction

Today flexible waveguide systems for ultrasonic vibrations transmission are found increasingly wide application in different areas of science and technology, e.g. ultrasonic thrombolysis [1,2], transurethral lithotripsy [3], fuel heating at low temperatures [4], remote actuation of ultrasonic motors [5], endoscopic neurosurgery [6], cleaning of hardly accessible channels in technical systems, etc. Unfortunately at present there are no methods for designing of such systems and their analysis and synthesis are implemented by empirical approach. Flexible waveguide vibrations are of complex nature and should be considered as coupled longitudinal–flexural vibrations. Investigation of such vibrations constitutes a sophisticated mathematical problem so primarily it seems reasonable to consider longitudinal and flexural vibration modes without regard to their interaction. It may be useful for determination of effective numerical and analytical methods for considered problem solution.

Coupled longitudinal–flexural vibrations of ultrasonic systems have been previously regarded in the research works by Zhou et al. [7,8] in which vibratory system consisting of a half-wave (in relation to the longitudinal vibration mode) horn and a transducer connected to it and comprising piezoelements generating longitudinal and flexural vibrations has been investigated. Flexural vibrations of horn with continuous variation of the cross-sectional area along the length are analyzed by slicing it into elementary sections with a constant cross-section and small length [8] with a subsequent application of the transfer matrices method. This involves multiplication of a large number of matrices and considerable computing time consumption.

An attempt of flexible waveguides modeling has been made in the research work by Bansevičius et al. [9] in which flexural vibrations of a waveguide with a constant cross-sectional area along the length have been considered. Unfortunately the represented results are well known from the classical theory of elastic rods vibrations and cannot be generalized in the case of waveguides with more complex law of variation of the cross-sectional area and centroidal moment of inertia along the length.

The problem of the flexible waveguides modeling is also considered in the article by Gavin et al. [10] who investigated the finite-element model of a waveguide immersed into fluid. Although the model enables investigation of the waveguides with arbitrary complex law of variation of the cross-sectional area and centroidal moment of inertia along the length, it is based on some assumptions reducing model's practical value. Particularly the problem is considered to be axisymmetric providing investigation of only

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Nomenclature

c	longitudinal thin-wire ultrasonic wave speed in the waveguide material, m/s
d	waveguide diameter, m
D_i	ith waveguide step diameter, m
E	modulus of elasticity of the waveguide material, Pa
f	vibration frequency, Hz
G	shear modulus of the waveguide material, Pa
J	centroidal moment of inertia of the waveguide cross-section, m^4
k	wave number value for the longitudinal vibration mode, m^{-1}
K_s	shape factor of the waveguide cross-section
L	total waveguide length, m
L_i	ith waveguide step length, m
S	waveguide cross-sectional area, m^2
x	longitudinal coordinate for the waveguide transitional section, m
x_i	longitudinal coordinate for the ith waveguide step, m
ΔL	length of the waveguide transitional section, m
α	amplitude of cross-section angular displacements for the waveguide transitional section, radian
α_i	amplitude of cross-section angular displacements for the ith waveguide step, radian
η	transverse displacement amplitude for the waveguide transitional section, m
η_i	transverse displacement amplitude for the ith waveguide step, m
κ	wave number value for the flexural vibration mode propagating in the waveguide transitional section, m^{-1}
κ_i	wave number value for the flexural vibration mode propagating in the ith waveguide step, m^{-1}
ν	Poisson's ratio of the waveguide material
ξ	longitudinal displacement amplitude for the waveguide transitional section, m
ξ_i	longitudinal displacement amplitude for the ith waveguide step, m
ρ	waveguide material density, kg/m^3
ω	circular vibration frequency, Hz

longitudinal vibrations. At the same time onset of flexural vibrations significantly reduces efficiency of ultrasound transmission along the long-length waveguides and therefore should be considered during their design.

2. Modeling technique

2.1. Waveguide design description

On the basis of empirical research the authors have determined and patented [2] rational geometric parameters of flexible waveguides for ultrasonic thrombolysis providing efficient transmission of longitudinal vibrations along the long-length waveguides (up to 900 mm) as well as such waveguides manufacturing technology. The diagram of two-step waveguide design is shown in Fig. 1.

Waveguide consists of two cylindrical sections (steps) 1 and 2 with a constant cross-section connected by the smooth transitional section 3 of Fourier horn type intended for reduction of stress concentration. Waveguide is manufactured from stainless steel rod by plasma-electrolytic etching and attached by soldering to the threaded joining element intended for connection of the waveguide with longitudinally vibrating horn.

To start with the waveguide vibration modeling it is essential to give mathematical description of the shape of the transitional section 3 between the steps 1 and 2. Let us suppose that the shape is specified by the diameter values d_i at the uniformly distributed points x_i, $i = 1, \ldots, N$ (in the example given below $N = 7$). Without limiting generality let us set coordinate origin in the input (proximal relative to the horn) cross-section of the transitional section, i.e. suppose $x_1 = 0$. Coordinate values x_i and corresponding diameter values d_i according to the patent [2] are given in Table 1.

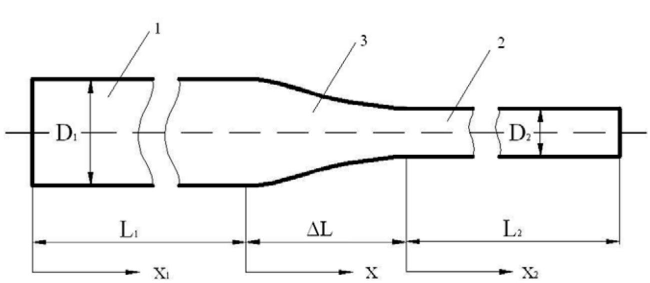

Fig. 1. Design of two-step waveguide.

The value d_1 corresponds to the diameter D_1 of the waveguide step with a larger cross-sectional area (input waveguide step 1) and d_N corresponds to the diameter D_2 of the step with a smaller cross-sectional area (output step 2). Let us approximate the shape of the transitional section 3 by the polynomial of the form $d(x) = \sum_{k=0}^{m} a_k x^k$. The curve defined by this polynomial should pass through the specified points of the shape which is equivalent to fulfillment of N conditions of the form $d(x_i) = d_i$. Moreover conditions of the smooth connection between the transitional section and waveguide steps should be satisfied. These conditions are described by two equations of the form $d'(0) = 0$ and $d'(x_N) = 0$. Thus in total amount $N + 2$ conditions should be fulfilled, i.e. polynomial degree m is determined by equation $m = N + 1$. We get $a_0 = D_1$ from the condition $d(0) = D_1$ and $a_1 = 0$ from the condition $d'(0) = 0$. For determination of the rest coefficients of the polynomial it is necessary to solve a system of N linear equations. As a result of calculation for the data given in Table 1 values of the coefficients given in Table 2 were obtained.

2.2. Resonance conditions for flexural vibrations

2.2.1. Euler–Bernoulli's theory

Supposing the input waveguide step 1 has the length L_1, the output step 2 has the length L_2 and the transitional section 3 has the length ΔL, let us determine lengths relation providing resonance of flexural waveguide vibrations for the given frequency f. For this purpose we will first consider flexural vibrations of the transitional section. Let us denote transverse displacement

Table 1
Shape of the transitional section.

Coordinate x, mm	0	1	2	3	4	5	6
Diameter d, mm	2	1.89	1.62	1.33	1.13	1	0.9

Table 2
Values of the coefficients.

a_2	a_3	a_4	a_5	a_6	a_7	a_8
−115.1	−741.2	6.2×10^6	-3.5×10^8	-2.0×10^{10}	-3.3×10^{13}	3.6×10^{15}