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Energy trapping in power transmission through a circular cylindrical elastic shell by finite piezoelectric transducers

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Abstract

We study the transmission of electric energy through a circular cylindrical elastic shell by acoustic wave propagation and piezoelectric transducers. Our mechanics model consists of a circular cylindrical elastic shell with finite piezoelectric patches on both sides of the shell. A theoretical analysis using the equations of elasticity and piezoelectricity is performed. A trigonometric series solution is obtained. Output voltage and transmitted power are calculated. Confinement and localization of the vibration energy (energy trapping) is studied which can only be understood from analyzing finite transducers. It is shown that when thickness-twist mode is used the structure shows energy trapping with which the vibration can be confined to the transducer region. It is also shown that energy trapping is sensitive to the geometric and physical parameters of the structure.

Keywords: Piezoelectricity; Vibration; Shell

1. Introduction

There has been recent interest in periodically charging and/or recharging batteries that power electronic devices operating in a sealed armor or other hazardous environments, such as nuclear storage facilities, into which the physical access is prohibited. For instance, to ensure the reliability and performance of nuclear stockpiles, there have been proposals that piezoelectric transducers are used to generate acoustic waves propagating through a sealed armor for transmitting a small amount of power to the electronic devices inside the sealed armor. In a few recent papers [1–6], the possibility of transmitting a certain amount of energy through a metal armor was explored by theoretical and experimental studies. The procedure involves the generation and propagation of acoustic waves and energy harvesting from the waves using piezoelectric transducers. Once the acoustic wave energy is harvested into electrical energy, batteries can be charged through circuit design [7–9]. This paper is mainly about the structural aspect of vibration mode distribution. Therefore the output circuit will be represented by an impedance [1]. The feasibility of this power transmission technique was demonstrated experimentally with 110 W power transmission at 88% efficiency [4]. There are also other possible applications of the technology like data transmission through a wall by acoustic waves.

The basic behaviors of the above power transmission procedure can be described by the theory of linear piezoelectricity. In Ref. [1], the armor through which energy is to be transmitted was idealized as an unbounded plate with two piezoelectric transducer layers covering the entire elastic plate. The whole structure vibrates in pure thickness modes without in-plane variations. Mathematically, the problem depends on one spatial variable, i.e., the plate thickness coordinate only. Therefore a simple, exact analysis was possible. More refined analyses of power transmission through an elastic plate were given in [10,11], where finite piezoelectric transducers were considered. It was shown in [10,11] that the vibration can be confined to the

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finite transducer region (called energy trapping) so that the whole plate structure does not vibrate globally which is desirable for practical purposes.

In a recent analysis [12], the armor was represented by a circular cylindrical shell which is a closed structure and is a more accurate representation of real situations. In the model of [12], the piezoelectric transducers in and out of the armor are also closed circular cylindrical shells and the whole structure vibrates in pure thickness modes with radial variations only. In this paper, we consider the more practical situation of a closed circular cylindrical elastic wall with finite piezoelectric patches on both sides. We will use the exact equations of linear elasticity and piezoelectricity to model the shell and the transducers, respectively. Although the finite element numerical method can be used to analyze almost all problems, it is not particularly efficient in analyzing the high-frequency thickness modes in this manuscript [13]. This is because thickness modes vary along the shell thickness. When finite element method is used, the element size is determined by the shell thickness and enough elements are needed along the thickness. Since element aspect ratio cannot be large for good numerical behavior, many elements will be needed for the whole structure. For the problem we want to analyze, a theoretical analysis can be performed which is usually desirable. Specifically, a trigonometric series solution is obtained with which we examine the effect of energy trapping in power transmission through a circular cylindrical shell.

2. Structure

Consider a circular cylindrical elastic shell of inner radius, b, and outer radius, c, as shown in Fig. 1. The shell is unbounded in the z-direction determined by the right-hand rule from the x- and y-axes in the figure. Only a cross-section of the shell is shown. We consider a unit thickness of the shell in the z-direction. Two piezoelectric transducers represented by the shaded areas are attached to the shell. The two radii going through the edges of the transducers form an angle 2β. There are no limitations on the magnitude of β except it has to be no larger than π. The transducers are made of ceramics poled in the z-direction. The inner transducer is electroded at $r = a$, b. The outer transducer is electroded at $r = c$, d. The four electrodes are shown by the thick lines in the figure. Under an applied voltage $2V_1$ across the outer transducer, the transducer and the entire structure are excited into a shear motion. The shell can be made from either a metal or a dielectric. When the shell is metallic, a very thin insulating later is assumed between the transducer electrodes and the shell. The insulating layer is very thin, its thickness and mechanical effects are neglected. $2V_1$ is a known, time-harmonic driving voltage. $2V_2$ is the unknown output voltage. I_1 and I_2 are input and output currents. Z is the impedance of the output circuit in time-harmonic motions. A cylindrical coordinate system is defined by $x = r\cos\theta$, $y = r\sin\theta$ and $z = z$. In the index notation below (x, y, z) correspond to $(1, 2, 3)$.

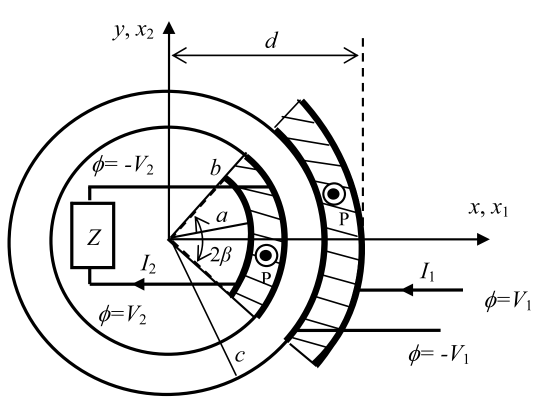

Fig. 1. A circular cylindrical elastic shell with two finite piezoelectric transducers.

In principle, both extensional and shear waves can be used for power transmission. The structure in Fig. 1 is for shear waves that can exist by themselves without coupling to other modes. The shear waves chosen have energy trapping by which the vibration can be confined to the region of the transducers. However, extensional modes in the length and thickness directions are inherently coupled [14]. Since length extensional modes are not trapped, they cause global vibration of the structure which is undesirable in the application considered.

3. Governing equations

For the material orientation and electrode configuration in Fig. 1, the shell can be electrically excited into the so-called axial thickness-shear, thickness-twist, anti-plane, or shear-horizontal mode with

$$u_1 = u_2 = 0, \quad u_3 = u(x_1, x_2, t), \tag{1}$$

where x_1, x_2 and x_3 correspond to x, y and z.

3.1. Equations for the elastic shell

Consider a shell of an isotropic elastic material. The non-vanishing components of the strain S_{ij} and stress T_{ij} are

$$\begin{aligned} &S_4 = 2S_{32} = u_{,2}, \quad S_5 = 2S_{31} = u_{,1}, \\ &T_4 = \mu u_{,2}, \quad T_5 = \mu u_{,1}, \end{aligned} \tag{2}$$

where μ is the shear elastic constant. The nontrivial equation of motion takes the following form:

$$c_2^2 \nabla^2 u = \ddot{u}, \tag{3}$$

where $\nabla^2 = \partial_1^2 + \partial_2^2$ is the two-dimensional Laplacian. c_2 is the speed of plane shear waves given by

$$c_2^2 = \mu/\rho_1, \tag{4}$$