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Spatially variant regularization of lateral displacement measurement using variance

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ABSTRACT

The purpose of this work is to confirm the effectiveness of our proposed spatially variant displacement component-dependent regularization for our previously developed ultrasonic two-dimensional (2D) displacement vector measurement methods, i.e., 2D cross-spectrum phase gradient method (CSPGM), 2D autocorrelation method (AM), and 2D Doppler method (DM). Generally, the measurement accuracy of lateral displacement spatially varies and the accuracy is lower than that of axial displacement that is accurate enough. This inaccurate measurement causes an instability in a 2D shear modulus reconstruction. Thus, the spatially variant lateral displacement regularization using the lateral displacement variance will be effective in obtaining an accurate lateral strain measurement and a stable shear modulus reconstruction than a conventional spatially uniform regularization. The effectiveness is verified through agar phantom experiments. The agar phantom [60 mm (height) × 100 mm (lateral width) × 40 mm (elevational width)] that has, at a depth of 10 mm, a circular cylindrical inclusion (dia. = 10 mm) of a higher shear modulus (2.95 and 1.43 × 10^6 N/m^2, i.e., relative shear modulus, 2.06) is compressed in the axial direction from the upper surface of the phantom using a commercial linear array type transducer that has a nominal frequency of 7.5-MHz. Because a contrast-to-noise ratio (CNR) expresses the detectability of the inhomogeneous region in the lateral strain image and further has almost the same sense as that of signal-to-noise ratio (SNR) for strain measurement, the obtained results show that the proposed spatially variant lateral displacement regularization yields a more accurate lateral strain measurement as well as a higher detectability in the lateral strain image (e.g., CNRs and SNRs for 2D CSPGM, 2.36 vs 2.27 and 1.74 vs 1.71, respectively). Furthermore, the spatially variant lateral displacement regularization yields a more stable and more accurate 2D shear modulus reconstruction than the uniform regularization (however, for the regularized relative shear modulus reconstructions, slightly accurate, e.g., for 2D CSPGM, 1.51 vs 1.50). These results indicate that the spatially variant displacement component-dependent regularization will enable the 2D shear modulus reconstruction to be used as practical diagnostic and monitoring tools for the effectiveness of various noninvasive therapy techniques of soft tissue diseases (e.g., breast, liver cancers). Application of the regularization to the elevational displacement will also increase the stability of a three-dimensional (3D) reconstruction.

1. Introduction

It is remarkable that the pathological state of human soft tissues highly correlates with their static and low-frequency mechanical properties, particularly, shear elasticity. Accordingly, many researchers including us have been developing strain [1–6] or displacement-measurement-based shear modulus reconstruction methods [7,8] and using various ultrasonic (US) strain/displacement measurement methods (e.g., Doppler method and autocorrelation method [9,10], cross-correlation method [11–13], sum-squared difference (SSD) method [14], our developed multidimensional cross-spectrum phase gradient method (MCSPGM), i.e., three-dimensional (3D) or 2D CSPGM [15,16], multidimensional autocorrelation method (MAM) [17,18], and multidimensional Doppler method (MDM) [17,18]).

For shear modulus reconstruction, various US methods have also been developed [1–4,7,8], in addition to magnetic resonance (MR)-based techniques [5,6] (i.e., the direct [1–6] and iterative [7,8] inversion approaches). In our case, for various *in vivo* tissues (e.g., breast [19,20] and liver [19]), we have been developing the direct 1D [1,19–21], 2D [1,19,22], and 3D [19,23,24] shear modulus reconstruction techniques as diagnostic and monitoring tools for the effectiveness of simple, minimally invasive therapy techniques (e.g., chemotherapy, cryotherapy, and thermal therapy) [19]. In the aforementioned reconstruction techniques, 1D, 2D, and 3D models are respectively used together with the 1D axial strain field, and the 2D and 3D strain tensor fields that are obtained by

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differentiating the measured 1D axial displacement field, and the 2D and 3D displacement vector fields (i.e., generated by extracorporeally applied pressure or vibration, or spontaneous heart motion). A relative shear modulus, but not an absolute modulus can also be obtained [25,22].

We earlier reported the effectiveness of regularization for stabilizing strain (displacement) measurement methods [26,27] and shear modulus reconstruction methods [26,28] (i.e., for decreasing the instabilities of the measurement and reconstruction due to contaminations in echo data). Such regularization enables the incorporation of the *a priori* knowledge that the targets are spatially smooth. Because the target distributions are respectively low-frequency deformations [29] and mechanical properties [30,31], the regularization also increases the accuracies of the measurement and reconstruction, and yields useful and unique results. Thus, the regularization solves the so-called ill-conditioned problems (i.e., nonexistence, nonuniqueness, and instability). In studies by other groups, the regularization is also applied to displacement measurement (using the optical flow algorithm) [32] and Young's modulus reconstruction (using the Newton–Raphson method) [7,8].

Regarding the regularization of the shear modulus reconstruction, we further proposed to properly set the regularization parameters at each position (i.e., spatially variant regularization parameters) [26,28], because the measurement accuracies of displacements and strains spatially vary. In other words, although the cross-validation method (i.e., using the independence of the raw vector of the matrix) is used in modulus reconstruction for a region of interest (ROI) (e.g., [7]), our measurement and reconstruction should be properly regularized at each position according to the measurement accuracies of displacements and strains, respectively. Because the measurement accuracies of strains can be evaluated using the correlation coefficient obtained after phase matching [15], the regularization parameters were set preliminarily proportional to the reciprocal of the power of the correlation coefficient [26]. However, the use of the variances of the strain tensor component measurements instead of the correlation coefficient was more effective in obtaining more uniformly stable, and more accurate results [28].

To achieve the more accurate shear modulus reconstruction, the higher dimensional shear modulus reconstruction should be performed rather than the lower dimensional shear modulus reconstruction (i.e., about 3D, 2D, and 1D reconstructions) [19,20,23]. For such multidimensional shear modulus reconstructions, when using displacement vector measurement methods (i.e., MCSPGM, MAM, and MDM) without multiple beamforming methods (i.e., multiple beam transmitting method [17,19], multiple directional synthetic aperture method [17,19] or lateral cosine modulation (LCM) method [33–36,17,19]), because the stabilities and accuracies of lateral/elevational displacement measurements are much lower than that of the axial displacement measurement [15–18], the regularization of lateral/elevational displacements is particularly effective (i.e., an our developed displacement component-dependent regularization) [26,27]. Generally, for both axial and lateral deformation cases, the axial displacement measurement is stable and accurate enough to perform a shear modulus reconstruction, i.e., 1D reconstruction [20,21]. Thus, for the axial displacement measurement, the regularization is not always required, although the regularization increases the stability and accuracy in the measurement absolutely [26,27]. However, for the multidimensional (2D or 3D) reconstruction, when special data acquisition systems that the multiple beamforming methods require are non-available, the regularization of lateral/elevational displacements is particularly effective.

Because the effectiveness of the spatially variant regularization using echo correlation coefficient on the displacement measurement has already been verified [26], in this report, the lateral displacement regularization using the lateral displacement variance is performed for MCSPGM, MAM, and MDM. The same agar phantom having a stiff cylindrical inclusion as that used in a uniform regularization of lateral displacement in Ref. [27] is used, and the effectiveness of the spatially variant regularization is verified by comparing the regularized results (Section 3). The effects of the spatially variant and uniform lateral displacement regularizations on the shear modulus reconstruction are also compared. Finally, conclusions are provided in Section 4.

2. Spatially variant, displacement component-dependent regularization

Using MCSPGM [15,16], MAM [17] or MDM [17], we obtain simultaneous equations for the unknown displacement vector $(\mathbf{u_x},\mathbf{u_y},\mathbf{u_z})^T$ at each position (i.e., one derived from the phase in (1) in Ref. [15] or Ref. [16], and (1) and (2) derived in Ref. [17]). By simultaneously dealing with the equations in the region of interest (ROI), we obtain the simultaneous equations for the unknown displacement vector distribution, i.e.,

$$\mathbf{Au} = \mathbf{b}$$

where $\mathbf{u}$ is expressed as the column vector composed of the axial, lateral, and elevational displacement component distributions $\mathbf{u_x}$, $\mathbf{u_y}$, and $\mathbf{u_z}$, i.e., $(\mathbf{u_x}^T, \mathbf{u_y}^T, \mathbf{u_z}^T)^T$, and $\mathbf{A}$ and $\mathbf{b}$ are respectively derived using the frequencies and phases.

By applying regularization [26,27] to the respective displacement component distributions $\mathbf{u_x}$, $\mathbf{u_y}$, and $\mathbf{u_z}$, we obtain the cost function $\mathrm{II}(\mathbf{u})$ that is minimized with respect to the displacement vector distribution $\mathbf{u}$.

$$\begin{aligned}\mathrm{II}(\mathbf{u}) = {} & ||\mathbf{b}-\mathbf{Au}||^2 + \alpha_{0x}||\mathbf{u_x}||^2 + \alpha_{0y}||\mathbf{u_y}||^2 + \alpha_{0z}||\mathbf{u_z}||^2 \\ & + \alpha_{1x}||\mathbf{Du_x}||^2 + \alpha_{1y}||\mathbf{Du_y}||^2 + \alpha_{1z}||\mathbf{Du_z}||^2 \\ & + \alpha_{2x}||\mathbf{D^TDu_x}||^2 + \alpha_{2y}||\mathbf{D^TDu_y}||^2 + \alpha_{2z}||\mathbf{D^TDu_z}||^2 \end{aligned} \quad (1)$$

where $\mathbf{D}$ and $\mathbf{D^TD}$, respectively, denote the gradient and Laplacian operators for the displacements, and α_{0x}, α_{0y}, α_{0z}, α_{1x}, α_{1y}, α_{1z}, α_{2x}, α_{2y}, and α_{2z} are regularization parameters. All the L_2-norms in right-hand side except for the first L_2-norms are referred to as penalty terms.

Although a regularization parameter can also be set to each L_2-norms of the displacement vector distribution $\mathbf{u}$ (i.e., $||\mathbf{u}||^2$), the gradient and Laplacian distributions (i.e., $||\mathbf{Du}||^2$ and $||\mathbf{D^TDu}||^2$ (i.e., penalty terms), because the measurement accuracies of the displacement component distributions differ from each other, particularly due to the differences between the instantaneous frequencies (i.e., US frequency and modulation frequencies) and bandwidths in the respective directions [15–18], the regularization should be properly performed with respect to the respective displacement component distributions $\mathbf{u_x}$, $\mathbf{u_y}$, and $\mathbf{u_z}$ using different regularization parameters, i.e., displacement component-dependent regularization parameters α_{0x}, α_{0y}, α_{0z}, α_{1x}, α_{1y}, α_{1z}, α_{2x}, α_{2y}, and α_{2z} [26,27]. However, more strictly, the regularization parameters should be properly set according to the measurement accuracies of the respective displacement components at each position, i.e., spatially variant, displacement component-dependent regularization.

The measurement accuracies can be evaluated at each position using the ultrasound parameters (frequencies and bandwidths), echo SNR or correlation coefficient (peak value of cross-correlation function) [26]. Although initially the reciprocal of the power of the correlation coefficient ρ is used in the experiments using an agar phantom and a human *invivo* liver [26], one approach to determining such regularization parameters is the use of the variances of