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ScienceDirect
Ultrasonics 44 (2006) 381–390
Ultrasonics
www.elsevier.com/locate/ultras

The wavelet response as a multiscale characterization of scattering processes at granular interfaces

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Received 30 August 2005; received in revised form 3 February 2006; accepted 21 May 2006
Available online 19 June 2006

Abstract

We perform a multiscale analysis of the backscattering properties of a complex interface between water and a layer of randomly arranged glass beads with diameter $D = 1$ mm. An acoustical experiment is done to record the wavelet response of the interface in a large frequency range from $\lambda/D = 0.3$ to $\lambda/D = 15$. The wavelet response is a physical analog of the mathematical wavelet transform which possesses nice properties to detect and characterize abrupt changes in signals. The experimental wavelet response allows to identify five frequency domains corresponding to different backscattering properties of the complex interface. This puts quantitative limits to the validity domains of the models used to represent the interface and which are flat elastic, flat visco-elastic, rough random half-space with multiple scattering, and rough elastic from long to short wavelengths respectively. A physical explanation based on Mie scattering theory is proposed to explain the origin of the five frequency domains identified in the wavelet response.

PACS: 43.60.+d; *43.40.L; 43.58.+z; 45.70.−n; 46.40.Cd; 46.65.+g

Keywords: Multiscale; Multiple scattering; Granular medium; Wavelet transform; Coda; Mie scattering

1. Introduction

The concept of interface plays a key role in many imaging methods using back-scattered waves, and most reflexion techniques (e.g. seismics) are based on models where quasi-homogeneous domains are separated by sharp interfaces where the physical properties of the medium change abruptly [24]. These models are used to reproduce the data as a superimposition of reflected echoes coming from interfaces whose number, position and strength are to be determined. The localisation of the interfaces needs an accurate large-scale velocity model, also called to macro-model, whose inversion is a highly non-linear tomography problem [32,18,6,4,19]. The parameters (i.e. compressional and shear velocities) of the macro-model are adjusted in order to fit with the arrival times. More sophisticated macro-models are eventually used to account for the attenuating visco-elastic properties of the media and the dispersion of the body waves [29,26]. The complete tomographic image is obtained by completing the macro-model with a small-scale distribution of impedance contrasts representing the interfaces producing the recorded echoes. In most approaches, abrupt impedance contrasts are represented by step-like functions (i.e. the Heaviside distribution) whose amplitude is adjusted to fit with the sign and the magnitude of the echoes [33,34]

Many situations exist where the model described above is insufficient to correctly account for the reality, and the sharp and step-like model may be inadequate to represent

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doi:10.1016/j.ultras.2006.05.212

the interfaces in heterogeneous material. Geophysical examples are volcanoes, the weathered subsurface [9,35], and the shallow layers of sediments forming the seafloor [8,3]. Ultrasonic imaging in both medical applications and non-destructive testing (NdT) are also faced with deficiencies of the step-like interface models at transitions involving small-scale heterogeneous media like bones, fat and composite material. A way to tackle with these difficulties is to use rough-interface models where the transition surface between homogeneous bodies with differing properties is given a rough topography [36] with possibly a fractal geometry [31,10]. However, rough-interface modelling does not generally consider the heterogeneous nature of the material on both sides of the transition, and further sophistications of the interface model would be necessary to properly account for the complex wave phenomena occurring in the vicinity of the interface. A number of questions then arise: In which wavelength range is the step-like concept still valid to represent the transition between domains of highly heterogeneous material? At which wavelengths does the step-like model cease to be valid? Which kind of more sophisticated models should be used instead?

In the present study, we present and discuss experimental results designed to provide both qualitative and quantitative data about the modelling of an interface formed by the transition between water and a dense layer of randomly arranged glass beads. The choice of glass beads ensures that multiple scattering is likely to occur as observed in volcanoes and granular seafloor. Both the experiments and the analysis are based on the *wavelet-response* method introduced by [20] who showed that the continuous wavelet transform obtained by convolving a signal with a family of constant-shape wavelets [14] may be physically extended to the wavelet response where a family of wavelets are propagated (i.e. NOT convolved) through the medium to be analysed. The wavelet response is equivalent to the wavelet transform of the reflectivity distribution when the first Born approximation is valid. Consequently, the interesting properties of the wavelet transform concerning the characterisation of abrupt changes in signals [22,12,1,2] are retrieved in the wavelet response which may be used to remotely analyse the multiscale structure of acoustical interfaces [37,13,39,38,23].

This paper continues with a brief presentation of the continuous wavelet transform and its extension to the wavelet-response method. Then, the experimental setup is detailed and the experimental wavelet response of the surface of a thick layer of glass bead is presented. In a next section, the wavelet response is analysed in the framework of the singularity characterisation toolbox developed for the continuous wavelet transform [22]. In a last section, we discuss the wave phenomena relevant in the different wavelength bands identified in the wavelet response of the interface. Our physical interpretation is based on the microscopic Mie scattering occurring in the heterogeneous medium.

2. Analysing method: the wavelet response

The wavelet response has been introduced in details by [21], and we only recall the main steps of its derivation from the classical continuous wavelet transform which consists in convolving a signal with a family of constant-shape analysing wavelets [1,14]. The wavelet family is obtained by dilating an analysing wavelet $g(t)$,

$$\mathscr{D}_a g(t) \equiv \frac{1}{a} g\left(\frac{t}{a}\right), \tag{1}$$

where $\mathscr{D}_a$ represents the dilation operator indexed by the dilation $a > 0$ which is inversely proportional to the frequency. The wavelets obtained for a dilation range $a_{\min} \leqslant a \leqslant a_{\max}$ have the same shape and constitute a wavelet family spanning a wide wavelength range well-adapted to study multiscale wave phenomena [20,21]. The analysing wavelet must be a time-localised oscillating function with a band-pass spectrum and, at least, a zero-order vanishing moment. For instance, the well-known Ricker source function [17,15] frequently used in seismic modelling is an acceptable analysing wavelet g.

The wavelet transform is obtained by convolving the whole wavelet family with the analysed signal, $s(t)$,

$$\mathscr{W}[g, s](b, a) \equiv (\mathscr{D}_a g * s)(b), \tag{2}$$

where $*$ is the convolution operator and b is a translation parameter. The main property needed in the present study is the covariance of the wavelet transform which indicates that the wavelet transform of a dilated function is the wavelet transform of the non-dilated function rescaled on both the a and b axes:

$$\mathscr{W}[g, \mathscr{D}_\beta s](b, a) = \frac{1}{\beta} \mathscr{W}[g, s]\left(\frac{b}{\beta}, \frac{a}{\beta}\right). \tag{3}$$

When applied to an homogeneous function of degree $\alpha \in R$ such that,

$$s(\beta x) = \beta^\alpha s(x), \tag{4}$$

Eq. (3) simplifies into,

$$\mathscr{W}[g, s](\beta b, \beta a) = \beta^\alpha \mathscr{W}[g, s](b, a), \tag{5}$$

which indicates that the whole wavelet transform of a homogeneous function can be obtained from the wavelet transform taken at a given dilation [22]:

$$\mathscr{W}[g, s](b, a') = \left(\frac{a'}{a}\right)^{1+\alpha} \mathscr{D}_{a'/a} \mathscr{W}[g, s](b, a), \tag{6}$$

where the dilation operator is understood to act on the translation variable b only. The geometrical sense of this equation is that the wavelet transform of a homogeneous singularity has the appearance of a cone whose apex points onto the singularity for $a \downarrow 0^+$. Eq. (6) indicates that the magnitude of the wavelet transform is $\propto a^\alpha$ when sampled along the cone lines also called the ridge functions (see [1,2,21] for details). The Heaviside distribution H used to represent step-like interfaces as discussed above is homoge-