Available online at www.sciencedirect.com

ScienceDirect

Ultrasonics 48 (2008) 85–90

Ultrasonics

www.elsevier.com/locate/ultras

Short Communication

The Rayleigh–Plesset equation in terms of volume with explicit shear losses

T.G. Leighton *

Institute of Sound and Vibration Research, University of Southampton, Highfield, Southampton SO17 1BJ, UK

Received 5 March 2007; accepted 25 October 2007
Available online 4 November 2007

Abstract

The most common nonlinear equation of motion for the damped pulsation of a spherical gas bubble in an infinite body of liquid is the Rayleigh–Plesset equation, expressed in terms of the dependency of the bubble radius on the conditions pertaining in the gas and liquid (the so-called 'radius frame'). However over the past few decades several important analyses have been based on a heuristically derived small-amplitude expansion of the Rayleigh–Plesset equation which considers the bubble volume, instead of the radius, as the parameter of interest, and for which the dissipation term is not derived from first principles. So common is the use of this equation in some fields that the inherent differences between it and the 'radius frame' Rayleigh–Plesset equation are not emphasised, and it is important in comparing the results of the two equations to understand that they differ both in terms of damping, and in the extent to which they neglect higher order terms. This paper highlights these differences. Furthermore, it derives a 'volume frame' version of the Rayleigh–Plesset equation which contains exactly the same basic physics for dissipation, and retains terms to the same high order, as does the 'radius frame' Rayleigh–Plesset equation. Use of this equation will allow like-with-like comparisons between predictions in the two frames.
© 2007 Elsevier B.V. All rights reserved.

PACS: 43.25.Yw; 43.25.Lj; 43.30.−k; 43.35.Ei

Keywords: Cavitation; Rayleigh–Plesset; Viscous damping; Bubbles; Parametric sonar; Difference frequency

1. Introduction

The most popular nonlinear equation for describing the nonlinear response of a gas bubble in liquid to a driving pressure field is the Rayleigh–Plesset equation. This can be derived from first principles using the bubble radius R as the dynamic parameter (which will here be termed the 'radius frame' approach):

$$R\ddot{R}+\frac{3\dot{R}^2}{2}=\frac{1}{\rho_0}\left(p_{\mathrm{L}}-\frac{4\eta\dot{R}}{R}-p_\infty\right) \quad (1)$$

where ρ_0 is the unperturbed liquid density, η is the shear viscosity of the liquid, and p_∞ is the liquid pressure far from the bubble, which is here assumed to consist of a static pressure p_0 and an applied acoustic field $P(t)$, such that $p_\infty = p_0 + P(t)$ [1]. When a polytropic gas law is used to evaluate the liquid pressure at the bubble wall (p_{L}), and the contributions of surface tension (σ) and vapour pressure (p_{v}) are included, Eq. (1) becomes

$$R\ddot{R}+\frac{3\dot{R}^2}{2}=\frac{1}{\rho_0}\left(\left(p_0+\frac{2\sigma}{R_0}-p_{\mathrm{v}}\right)\left(\frac{R_0}{R}\right)^{3\kappa}+p_{\mathrm{v}}-\frac{2\sigma}{R}-\frac{4\eta\dot{R}}{R}-p_0-P(t)\right) \quad (2)$$

where R_0 is the unperturbed bubble radius. It is noted that use of the polytropic index (κ) adjusts the gas stiffness for reversible heat flow across the bubble wall, but does not describe any net thermal losses. The only dissipation present in (2) occurs through viscous losses.

However there exist heuristic formulations based on a form of the Rayleigh–Plesset equation in which the bubble

* Tel.: +44 2380 592291; fax: +44 2380 593190.
E-mail address: tgl@soton.ac.uk

0041-624X/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.ultras.2007.10.004

volume V is used as the dynamic parameter (which will be termed the 'volume frame' approach), where the damping is not derived from first principles. Furthermore, the 'volume frame' form of the damped Rayleigh–Plesset equation which is commonly quoted neglects higher order terms which are present in the 'radius frame' version (2). Therefore the two equations are not equivalent on two counts.

The predictions of the two approaches do not always agree, and before any differences can be attributed to another factor, it is important to ensure that one is comparing 'like-with-like'. It is not immediately apparent that this is being done, given the differences in the way dissipation is formulated, and the manner in which higher order terms are neglected in the 'volume frame' form which is widely used [2]. Therefore this study was undertaken to derive a 'volume frame' form of the Rayleigh–Plesset equation where the physics describing the dissipation is identical to that used when the Rayleigh–Plesset equation is cited in the radius frame (2), and where the higher order terms have not been assumed to be negligible.

This paper will proceed by using the following common assumptions: The bubble exists in an infinite medium. The bubble stays spherical at all times during the pulsation. Spatially uniform conditions exist within the bubble. The bubble radius is much smaller than the wavelength of the driving sound field. There are no body forces acting (e.g. gravity). Bulk viscous effects can be ignored. The density of the surrounding fluid is much greater than that of the internal gas. The gas content is constant.

2. Background

Of the various 'volume frame' equations for bubble dynamics [3], the form given by Zabolotskaya and Soluyan [2] has been most valuable and influential, and featured as the starting point in several notable studies. These include the bubble-mediated generation of difference frequencies when bubbles are insonified by two acoustic frequencies for a range of purposes, including bubble detection [4], the use of bubbles to enhance parametric sonar [5,6], and the acoustic characterization of gassy seabeds [7]. Biomedical investigations which have used the 'volume frame' include studies of contrast agent [8] and HIFU [9]. If the predictions of these important 'volume frame' studies are to be reconciled with those obtained using the 'radius frame' Rayleigh–Plesset Eq. (2), it is important to ensure that the comparison is of 'like-with-like', specifically that the equations of motion in each case contain the same physics and the same degree of approximation. This is the purpose of this paper.

The influential analysis of Zabolotskaya and Soluyan [2], which underpins the majority of studies of nonlinear bubble dynamics in the 'volume frame', begins with a statement (not derived) of the Rayleigh equation in the volume frame. The Rayleigh equation is the undamped form of the Rayleigh–Plesset equation, and the volume frame description given by Zabolotskaya and Soluyan [2] is

$$\left(\frac{\ddot{V}}{V^{1/3}}-\frac{\dot{V}^2}{6V^{4/3}}\right)=\frac{4\pi}{\rho_0}\left(\frac{3}{4\pi}\right)^{1/3}(p_g-p_\infty) \tag{3}$$

where p_g is the pressure in the bubble gas (assumed to be air in [2]). Understandably for the time, given the limited computing abilities then available, Zabolotskaya and Soluyan do not calculate output from this equation directly, but rather proceeded to generate a small amplitude expansion based on volume perturbations $V_\varepsilon(t)$ about an equilibrium bubble volume V_0

$$V=V_0+V_\varepsilon(t) \quad V_\varepsilon \ll V_0 \tag{4}$$

with an adiabatic gas law

$$p_g=p_0(V_0/V)^\gamma \tag{5}$$

where γ is the ratio of the specific heat capacity of the gas at constant pressure, to its value at constant volume. The effects of surface tension and vapour pressure are neglected. This expansion generated the following expression

$$\begin{aligned}&\ddot{V}_\varepsilon+\omega_M^2V_\varepsilon-\alpha_{ZS}V_\varepsilon^2-\beta_{ZS}(2\ddot{V}_\varepsilon V_\varepsilon+\dot{V}_\varepsilon^2)+F_{ZS}\dot{V}_\varepsilon\\&\quad=-\left(\frac{4\pi R_0}{\rho_0}\right)P(t)\end{aligned} \tag{6}$$

where $V_\varepsilon(t)$ is the perturbation in bubble volume, and where

$$\omega_M=\sqrt{\frac{3\gamma p_0}{\rho_0R_0^2}} \tag{7}$$

is the Minnaert frequency of the bubble and the parameters α_{ZS} and β_{ZS} represent the following groupings

$$\begin{aligned}\alpha_{ZS}&=3\beta_{ZS}(\gamma+1)\omega_M^2\\\beta_{ZS}&=1/(8\pi R_0^3)\end{aligned} \tag{8}$$

The term F_{ZS} was introduced in an *ad hoc* fashion to include dissipation. It was assumed to be frequency dependent.

The achievement of Zabolotskaya and Soluyan in generating this analysis should not be underestimated. Its timing perceptively heralded and facilitated a wealth of investigations which employed their findings (ranging from biomedical therapy to seabed exploration [4–9]), yet did so in a way which provided equations that were appropriate not only for the computing power of the day, but also over the decades that followed. Furthermore, this analysis provided a framework in which the physical influences of the various terms are transparent.

Over the thirty years and more since Eq. (6) was published by Zabolotskaya and Soluyan, its popularity has increased. It is now important to revisit the assumptions inherent in the formulation, and ask whether the assumptions required for its derivation in 1973 are still necessary, given increased computing power, and to highlight the implications of the continued use of those assumptions. This is particularly so in light of two issues, both of which relate to the impression which can be given that Eq. (6) is

Download English Version:

https://daneshyari.com/en/article/1760033

Download Persian Version:

https://daneshyari.com/article/1760033

Daneshyari.com