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# Love wave propagation in functionally graded piezoelectric material layer

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**Abstract**

An exact approach is used to investigate Love waves in functionally graded piezoelectric material (FGPM) layer bonded to a semi-infinite homogeneous solid. The piezoelectric material is polarized in $z$-axis direction and the material properties change gradually with the thickness of the layer. We here assume that all material properties of the piezoelectric layer have the same exponential function distribution along the $x$-axis direction. The analytical solutions of dispersion relations are obtained for electrically open or short circuit conditions. The effects of the gradient variation of material constants on the phase velocity, the group velocity, and the coupled electromechanical factor are discussed in detail. The displacement, electric potential, and stress distributions along thickness of the graded layer are calculated and plotted. Numerical examples indicate that appropriate gradient distributing of the material properties make Love waves to propagate along the surface of the piezoelectric layer, or a bigger electromechanical coupling factor can be obtained, which is in favor of acquiring a better performance in surface acoustic wave (SAW) devices.


*Keywords:* Love waves; Functionally graded piezoelectric material; Surface acoustic waves; Dispersion relation

## 1. Introduction

A new-style material called functionally graded material (FGM) was proposed to solve problems in the thermal-protection systems of aerospace structures in 1980s. Since then, FGM has attracted interest of investigators from many engineering disciplines. Today, FGM can be used not only in thermal-protection systems but also in electronic and many other fields. The results obtained for the FGM layered structures lead us to consider that the FGM may be applicable to surface acoustic wave (SAW) devices, if functionally graded piezoelectric material (FGPM) can be properly manufactured, as known from recent techniques for fabricating FGPMs [1].

Since White [2] invented the interdigital transducers (IDTs) utilized for transmitting and receiving SAW signals in 1965, SAW are adopted successfully in the electronic industry with filters, delay lines, resonators, and oscillators for signal processing applications. With the development of the material technology, FGPMs can be manufactured and used in SAW devices to improve the efficiency and other features. Hence, the research of wave propagation behaviors and characteristics in FGPM has become a topic of practice importance [2–11]. Liu and Tani investigated surface waves in FPGM plates with the application of strip element method [2–8]. Han et al. introduced a hybrid numerical method (HNM) to analyze characteristics of waves and transient responses in FGM cylinders [9,10]. Recently Han and Liu investigated the frequency and group velocity dispersion behaviors, and characteristic surfaces of waves in FGPM cylinders using an analytical numerical method [11]. Li et al. studied the behaviors of Love waves in a layered functionally graded piezoelectric structure using WKB method [12].

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Table 1
Material constants

| | $c_{44}(10^9 N/m^2)$ | $e_{15}(C/m^2)$ | $\varepsilon_{11}$ $(10^{-9}F/m)$ | $\rho(10^3kg/m^3)$ |
|---|---|---|---|---|
| PZT-5H | 23 | 17 | 15 | 7.5 |
| $SiO_2$ | 31.2 | 0 | 0.033 | 2.2 |

The layered structures with inhomogeneous boundary conditions, for example, a thin film on a substrate, are currently adopted to achieve high performance for SAW devices. It is well known that phase velocities and dispersion relations of acoustic waves are important for applications. In 1911, Love [13] analyzed a layered structure consisting of an isotropic elastic layer on an isotropic substrate with perfect bonding at the interface. He concluded that shear surface waves propagate in the layer and attenuate along thickness of the substrate if the velocity of the bulk shear wave in the layer is less than that in the substrate. These shear surface waves are now known as the Love waves and their polarization is perpendicular to the sagittal plane formed by both the normal to the interface of a medium and the wavevector in the direction of wave propagation [14]. Numerous investigations have been undertaken for the analysis of Love waves in piezoelectric media [15–18].

In this paper, we analyze the propagation of Love waves over a half space of elastic solid covered by a piezoelectric layer of finite thickness. The piezoelectric material is polarized in $z$-axis direction and the material properties change gradually with the thickness of the layer. We here assume that all material properties of the piezoelectric layer have the same exponential function distribution along the $x$-axis direction. The analytical solution of dispersion relations can be obtained for electrically open or short circuit conditions.

## 2. Problem formulation

Consider an anisotropic semi-infinite elastic substrate covered by a functionally graded piezoelectric material layer as illustrated in Fig. 1. The piezoelectric material is polarized in $z$-axis direction and the material properties change gradually with the thickness of the layer. For the piezoelectric layer, the equilibrium equations of elasticity without body forces and the Gauss' law of electrostatics without free charge are given as follows:

$$\sigma_{ji,j} = \rho \ddot{u}_i, \quad D_{i,i} = 0, \qquad i,j = 1,2,3, \tag{1}$$

where $\sigma_{ij}$ is the stress tensor, $D_i$ is the electric displacement, and $\rho$ is the mass density of the piezoelectric material.

On the assumption that the Love waves propagate in the $y$ direction, so the total out-of-plane displacement and the electric potential are expressed as

$$u = v = 0, \qquad w = w(x,y,t), \qquad \phi = \phi(x,y,t). \tag{2}$$

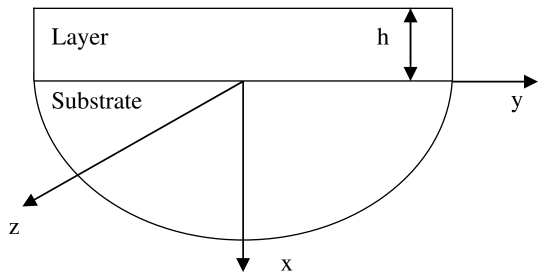


Fig. 1. An elastic half-space covered by a piezoelectric layer.

Substituting Eq. (2) into Eq. (1), we can obtain

$$\frac{\partial\tau_{zx}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} = \rho\frac{\partial^2 w}{\partial t^2}, \quad \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0. \tag{3}$$

The constitutive equations of the functionally gradient piezoelectric materials can be written as

$$\begin{aligned}\tau_{yz} &= c_{44}(x)\frac{\partial w}{\partial y} + e_{15}(x)\frac{\partial\phi}{\partial y},\\ \tau_{zx} &= c_{44}(x)\frac{\partial w}{\partial x} + e_{15}(x)\frac{\partial\phi}{\partial x},\end{aligned} \tag{4}$$

$$\begin{aligned}D_x &= e_{15}(x)\frac{\partial w}{\partial x} - \varepsilon_{11}(x)\frac{\partial\phi}{\partial x},\\ D_y &= e_{15}(x)\frac{\partial w}{\partial y} - \varepsilon_{11}(x)\frac{\partial\phi}{\partial x}.\end{aligned} \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (3), the governing equations of the piezoelectric layer are obtained as follows:

$$\begin{aligned}&\frac{\partial c_{44}(x)}{\partial x}\frac{\partial w}{\partial x} + c_{44}(x)\frac{\partial^2 w}{\partial x^2} + \frac{\partial e_{15}(x)}{\partial x}\frac{\partial\phi}{\partial x} + e_{15}(x)\frac{\partial^2\phi}{\partial x^2}\\ &\quad + c_{44}(x)\frac{\partial^2 w}{\partial y^2} + e_{15}(x)\frac{\partial^2\phi}{\partial y^2} = \rho\frac{\partial^2 w}{\partial t^2},\end{aligned} \tag{6}$$

$$\begin{aligned}&\frac{\partial e_{15}(x)}{\partial x}\frac{\partial w}{\partial x} + e_{15}(x)\frac{\partial^2 w}{\partial x^2} - \frac{\partial\varepsilon_{11}(x)}{\partial x}\frac{\partial\phi}{\partial x} - \varepsilon_{11}(x)\frac{\partial^2\phi}{\partial x^2}\\ &\quad + e_{15}(x)\frac{\partial^2 w}{\partial y^2} - \varepsilon_{11}(x)\frac{\partial^2\phi}{\partial y^2} = 0.\end{aligned} \tag{7}$$

We here assume that all material properties of the piezoelectric layer as Fig. 1 have the same exponential function distribution along the $x$-axis direction. Though these material constants distributions are unrealistic, it would allow us to comprehend the influence of material gradient upon the characteristics of wave propagation, and make use of it for designing more effective devices in practice. In the mean time, we can obtain one analytical and exact resolution with the assuming of exponential function distribution, which can be used to verify the accuracy of other numerical methods. The material properties are given as

$$\begin{aligned}c_{44}(x) &= c_{44}^0 e^{\beta x}, \qquad e_{15}(x) = e_{15}^0 e^{\beta x},\\ \varepsilon_{11}(x) &= \varepsilon_{11}^0 e^{\beta x}, \qquad \rho(x) = \rho^0 e^{\beta x},\end{aligned} \tag{8}$$