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Advances in Space Research 58 (2016) 950–966

ADVANCES IN SPACE RESEARCH (a COSPAR publication)

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Maximum collision probability considering variable size, shape, and orientation of covariance ellipse

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Received 2 December 2015; received in revised form 25 May 2016; accepted 29 May 2016
Available online 3 June 2016

Abstract

When engaging in the maximum collision probability ($P_{c\max}$) analysis for short-term conjunctions between two orbiting objects, it is important to clarify and understand the assumptions for obtaining $P_{c\max}$. Based on Chan's analytical formulae and analysis of covariance ellipse's variation of orientation, shape, and size in the two-dimensional conjunction plane, this paper proposes a clear and comprehensive analysis of maximum collision probability when considering these variables. Eight situations will be considered when calculating $P_{c\max}$ according to the varied orientation, shape, and size of the covariance ellipse. Three of the situations are not practical or meaningful; the remaining ones were completely or partially discussed in some of the previous works. These situations are discussed with uniform definitions and symbols and they are derived independently in this paper. The consequences are compared and validated by the results from previous works. Finally, a practical conjunction event is presented as a test case to demonstrate the effectiveness of methodology. Comparison of the $P_{c\max}$ presented in this paper with the empirical results from the curve or surface calculated by numerical method indicates that the relative error of $P_{c\max}$ is less than 0.0039%.

Keywords: Conjunction assessment; Maximum collision probability; Covariance ellipse; Orientation; Shape; Size

1. Introduction

As the number of on-orbit satellites and space debris continuously increases, the conjunction assessment and collision avoidance has become of increased concern. The closest approach distance and the collision probability are two important criteria in conjunction risk assessment. Currently, the main criteria is collision probability (P_c) between two objects in the conjunction.

For fixed miss distance and space objects' sizes, there is a maximum collision probability ($P_{c\max}$) that varies with size, shape, and orientation of error covariance ellipsoid. The estimation of $P_{c\max}$ is significant in the conjunction risk assessment. When the covariances of one or both objects are not known or unreliable, $P_{c\max}$ could be useful as an indicator of risk to assess the conjunction in the worst-case.

$P_{c\max}$ could also be used in the pre-filter of dangerous object if the computation time for $P_{c\max}$ is considerably less than the computation of P_c. Actually, as will be seen in this paper, $P_{c\max}$ can be computed rapidly by analytical or approximate expressions, while the more time-consuming

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http://dx.doi.org/10.1016/j.asr.2016.05.050

refined collision probability analysis always includes integral or iteration of series. If $P_{c\max}$ is below the predefined probability threshold, no further refined calculations are needed. As a guideline, the probability threshold should be less than the "yellow" threshold to avoid missing alarm, which is 10^{-5} for the space shuttle (Leleux et al., 2002). Therefore, this effectively decreases the computational time in using the pre-filter because of the larger number of conjunctions to be considered.

$P_{c\max}$ is normally larger than the real risk. The maximum P_c may not provide a definitive answer to the question of whether or not a risk mitigation maneuver is required (Frisbee, 2015). If the calculated maximum P_c is less than the action threshold, no risk mitigation action is necessary, then a useful final result is obtained. However, if the maximum P_c does violate an action threshold, it cannot be concluded that some type of risk mitigation action is absolutely necessary because the actual risk might still be orders of magnitude below that threshold.

$P_{c\max}$ can also determine the distance threshold in the pre-filter of a threatening object. $P_{c\max}$ and corresponding error variance can determine orbital prediction accuracy required for conjunction assessment that will prevent or minimize dilution of the probability calculations. (Alfano, 2003, 2004; Gottlieb et al., 2001; Peterson, 2004; Jenkin, 2004).

Mathematically, an ellipsoid in three-dimension space or an ellipse in two-dimension plane is characterized by its ***size***, ***shape***, and ***orientation***, as illustrated in Fig. 1. When the covariance ellipsoids are obtained from independently tracked measurements, the P_c is determined based on the "combined" covariance ellipse's size, shape, and orientation associated with each of the objects. The calculated P_c is affected by these three aspects simultaneously. The variation of one or all of them yields a maximum probability. Therefore, when dealing with the maximum collision probability problem it is significant to clarify and understand what assumptions are made to obtain $P_{c\max}$. Totally different results will be obtained with respect to different definition of $P_{c\max}$.

In practical conjunction assessment, the predicted orientation of covariance ellipse is generally of high accuracy. That is because the major axis of covariance ellipsoid is usually aligned very closely with the velocity of orbiting object, so the orientation of covariance ellipse in the "encounter" or "conjunction" plane is mainly determined by the conjunction geometry which is often believed to be accurate if the bias of propagation of orbits is ignored. Some previous works treated the orientation as a constant when analyzing the maximum probability (Alfriend et al., 1999; Chan, 2008). However, the predicted size and shape of covariance ellipse are generally less accurate, especially the former. Almost all works treated the size and shape as variables when handling with the maximum probability.

Alfriend et al. (1999) assumed that the probability density function (pdf) is constant over the sphere, and equal to the value at the center of the sphere. Based on this assumption, the value of covariance's scaling factor that maximizes P_c and the corresponding value of $P_{c\max}$ are presented. In this method the size represented by the scaling factor was the only variable, the shape and orientation were fixed.

Alfano (2003) developed a method to map regions of maximum probability for various satellite sizes, encounter geometries, and covariance sizes and shapes. Those regions were then examined to assess probability dilution. Charts were created to show the effects of positional uncertainty on probability calculation and assess probability dilution, to determine ephemeris accuracy requirements, and to establish distances for probability-based keep-out zones. In this method the size (1-sigma combined positional deviation), shape (aspect ratio), and orientation were taken into consideration to achieve $P_{c\max}$. The drawback is that no close-formed solution was given.

Alfano (2004) showed how to calculate the upper bounds of probability by determining the "worst" possible covariance parameters and orientation under some extreme assumptions when the aspect ratio of the combined covariance approaches infinity, as well as the major axis of the combined covariance ellipse aligned with the relative position vector. The upper bound of P_c in this extreme situation is often overestimated, which may be too conservative to serve as the discriminator for pre-filtering. As shown in this paper, the major axis of the covariance ellipse aligned with the relative position vector is an inevitable result if the variation of orientation is taken into account. This method provides the formulae of $P_{c\max}$ in that extreme situation.

Klinkrad (2006) combined two covariances and then scaled the combined covariance to find the maximum pdf

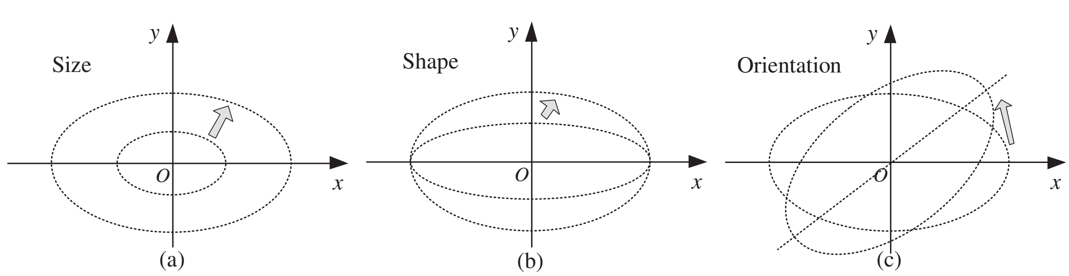

Fig. 1. The variation of ellipse's (a) size, (b) shape, and (c) orientation in two-dimensions plane.