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ScienceDirect

Advances in Space Research 58 (2016) 1146–1154

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

www.elsevier.com/locate/asr

Dispersive and dissipative nonlinear structures in degenerate Fermi–Dirac Pauli quantum plasma

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Received 14 October 2015; received in revised form 30 May 2016; accepted 31 May 2016
Available online 6 June 2016

Abstract

We study the interplay between dispersion due to the electron degeneracy parameter and dissipation caused by plasma resistivity, in degenerate Fermi–Dirac Pauli quantum plasma. Considering relativistic degeneracy pressure for electrons, we investigate both arbitrary and small amplitude nonlinear structures. The corresponding trajectories are also plotted in the phase plane. The linear analysis for the dispersion relation yields interesting features. The present work is anticipated to be of physical relevance in the study of compact magnetized astrophysical objects like white dwarfs.

Keywords: Fermi–Dirac quantum plasma; Relativistic degeneracy parameter; Quantum magnetohydrodynamics; Nonlinear structures

1. Introduction

Investigations of degenerate quantum plasmas have gained much interest in recent times, on account of their applications in super dense astrophysical environments, modern lasers and also in microelectronic devices. With very high particle number densities $\approx 10^{30} - 10^{36}\ \mathrm{m}^{-3}$, and low temperatures, quantum effects dominate in such environments. This occurs both in artificially created plasmas as in nanostructured quantum wells, quantum wires and quantum dots (Haug and Koch, 2004), and in nature as compact astronomical objects like white dwarfs (Balberg and Shapiro, 2000, Shah et al., 2011; Brodin and Marklund, 2007; Mushtaq and Qamar, 2009; Moslem et al., 2007). The inter-fermion distance being smaller than the thermal de Broglie wavelength $\lambda_B = h/(2\pi m_e k_B T)^{1/2}$ in such cases, along with the electron degeneracy (a consequence of Pauli exclusion principle) and tunneling effects, gives rise to new collective phenomena. Assuming a quantum hydrodynamic model for such quantum plasmas, various authors have shown that the delicate interplay between dissipation and dispersion leads to a variety of nonlinear structures like solitons, damped oscillations and shocks (Akbari-Moghanjoughi, 2011a; Marklund et al., 2007). Dense astronomical objects like white dwarfs can be treated as zero temperature degenerate Fermi gas because of the immense inward gravitational pressure, which can lead to the state of relativistic degeneracy for electrons (Chandrasekhar, 1939). The quantum Bohm force, on the other hand, is affected by the relativistic electron degeneracy, since the relativistic electron momentum is directly coupled to the plasma mass density via the relativistic degeneracy parameter (Akbari-Moghanjoughi, 2013). As the degeneracy state changes from nonrelativistic to

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http://dx.doi.org/10.1016/j.asr.2016.05.054
0273-1177/

relativistic limit e.g., in a white dwarf, the nonlinear dynamics of the quantum plasma also gets modified.

Some remarkable properties of Fermi–Dirac Pauli quantum plasmas were discussed in many recent works (Brodin and Marklund, 2007; Akbari-Moghanjoughi, 2011a; Ali et al., 2007; Aoutou et al., 2012; Asenjo et al., 2012;Khan, 2012; Li and Han, 2014;Masood et al., 2010; Wang et al., 2013; Shukla and Eliasson, 2010). In the present work, we shall study the interplay between dispersion due to the relativistic degeneracy pressure for the electrons and dissipation caused by plasma resistivity in nonlinear structures in Fermi–Dirac-Pauli quantum plasmas. It is this degeneracy pressure that provides the restoring force while the ions provide the inertia for the electrostatic shocks. The authors in Wang et al. (2013) considered a similar system to carry out the modulational instability of spin modified quantum magnetosonic waves, employing the nonlinear Schrödinger equation. The Sagdeev pseudopotential approach was used to investigate the propagation of arbitrary amplitude nonlinear ion waves in a relativistically degenerate pair plasma in the framework of quantum hydrodynamics model (Akbari-Moghanjoughi, 2011b,c). However, our present study is quite different from either of these works as we study the dynamical behaviour of the quantum magnetosonic waves for both arbitrary and small amplitudes. Since phase portraits play a significant role in studying the stability behaviour of dynamical systems, even in the absence of actual solutions, we also plot the trajectories in the phase plane (El et al., 2015).

The organization of the article is as follows. The Introduction is followed by the basic quantum plasma equations in Section 2. Linear analysis for the dispersion relation in carried out in Section 3. The arbitrary amplitude nonlinear structures, along with phase portraits are plotted in Section 4. For small amplitude analysis, reductive perturbation technique is applied in Section 5 to obtain the KdVB equation. Finally, Section 6 is kept for concluding remarks and summary.

2. Basic governing equations

To make the paper self-contained, we start by writing the basic governing equations for a collisionless, quasineutral ($n_e \approx n_i$), spin magnetized, perfectly degenerate superdense plasma, containing non degenerate cold ions and relativistically degenerate electrons, assuming the magnetohydrodynamic model for Fermi–Dirac Pauli quantum plasma (QMHD). It is worth mentioning here that high particle number density and low particle temperature in many compact interstellar objects make them a highly appropriate system for studying quantum plasmas. Matter exists in extreme conditions in such an environment, in the degenerate state. The plasma particles are randomly oriented in a non uniform magnetic field. However, in thermodynamic equilibrium, there is plasma magnetization in the direction of the external magnetic field **B**, due to the alignment of the electron spin with this field (Wang et al., 2013). Furthermore, we ignore the relativistic effects in the electron inertia. The only relativistic effect on the electrons is in the relativistic degeneracy pressure law. Additionally, we ignore the quantum effects for the ions due to their large mass compared to the electron mass. Neglecting the ion pressure in comparison with the electron degeneracy pressure, the closed set of quantum magnetohydrodynamics equations governing the dynamics of spin-induced magnetosonic waves, taking into account the quantum tunneling and spin-1/2 effects, in the center of mass frame, is given by the following equations (Wang et al., 2013):

The equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}$$

the momentum equation,

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{v} = \mathbf{j}\times\mathbf{B} - \nabla P_e + \mathbf{F}_Q \tag{2}$$

and the generalized Faraday law (Brodin and Marklund, 2007),

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\mathbf{v}\times\mathbf{B} - \frac{\mathbf{j}\times\mathbf{B}+\mathbf{F}_Q}{en} - \eta\mathbf{j} - \frac{m}{e^2\mu_0}\left(\frac{\partial}{\partial t} - \frac{\nabla\times\mathbf{B}}{en\mu_0}\cdot\nabla\right)\frac{\nabla\times\mathbf{B}}{n}\right] \tag{3}$$

It is to be noted here that non adiabatic effects such as viscosity may be neglected if we consider ideal gas dynamics (Camenzind et al., 2007). Our primary consideration in this work is to study the effect of plasma resistivity in dense astrophysical objects. The effect of viscosity will be taken up in a future work. The second point worth mentioning here is that the electron gas is assumed to be paramagnetic so that $\mu_B B \ll k_B T$. A dense plasma is usually characterized as cold and degenerate such as that encountered in metals and semiconductors. However, hot fusion plasma such as that found in dense stellar objects like white dwarfs may also be considered as quantum degenerate plasmas (Akbari-Moghanjoughi, 2010). In the above equations, the subscripts e and i referring to electron and ion, with m, n, e denoting mass, density and charge of the particles. The rest of the parameters have the following meanings:

$\rho = m_e n_e + m_i n_i$ is the total mass density,
$\mathbf{v} = (m_i n_i \mathbf{v}_i + m_e n_e \mathbf{v}_e)/\rho$ is the fluid velocity,
$\mathbf{j} = -\mu_0^{-1}\nabla\times(\mathbf{B}-\mu_0\mathbf{M})$ is the free current,
$\mathbf{F}_Q = \frac{\rho\rho_0\hbar^2}{2m_e m_i}\nabla\left(\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}\right) + M\nabla B$ is the quantum force,
μ_0 is the vacuum magnetic permeability,
M is the magnetization density,
η is the resistivity,
P_e is the electron degeneracy pressure,
ρ_0 is the equilibrium value of total mass density.

In deriving Eq. (3), certain approximations were used: e.g. we have neglected the displacement current, assumed the