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State dependent model predictive control for orbital rendezvous using pulse-width pulse-frequency modulated thrusters

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Abstract

This paper studies the pulse-width pulse-frequency modulation based trajectory planning for orbital rendezvous and proximity maneuvering near a non-cooperative spacecraft in an elliptical orbit. The problem is formulated by converting the continuous control input, output from the state dependent model predictive control, into a sequence of pulses of constant magnitude by controlling firing frequency and duration of constant-magnitude thrusters. The state dependent model predictive control is derived by minimizing the control error of states and control roughness of control input for a safe, smooth and fuel efficient approaching trajectory. The resulting nonlinear programming problem is converted into a series of quadratic programming problem and solved by numerical iteration using the receding horizon strategy. The numerical results show that the proposed state dependent model predictive control with the pulse-width pulse-frequency modulation is able to effectively generate optimized trajectories using equivalent control pulses for the proximity maneuvering with less energy consumption.

Keywords: Pulse-width pulse-frequency modulation; State dependent model predictive control; Autonomous rendezvous; Trajectory planning; Quadratic programming

1. Introduction

Autonomous rendezvous and proximity operations (ARPO) between a chaser spacecraft (chaser) and a non-cooperative spacecraft (target) have attracted extensive attentions from researchers concerning the autonomous active space debris removal and on-orbit servicing (Goodman, 2006; Gill et al., 2007; Subbarao and Welsh, 2008). Challenges arise as the chaser approaching the non-cooperative spacecraft or space debris to avoid the collision with the target in the ARPO with less fuel consumption and small thrusters. Therefore, stringent operational requirements are imposed on the trajectory planning and tracking control of the chaser to avoid the sudden change of control force that may cause the chattering of trajectory (Lembeck and Prussing, 1993; Guelman and Aleshin, 2001; Aghili, 2009; Gao et al., 2014; He et al., 2015).

Many control methodologies or algorithms have been developed to generate optimal approaching trajectories with the objectives of minimum propellant consumption, shortest approaching time, high control accuracy or the combinations of these effects, e.g., the mixed-integer linear programming (Richards et al., 2002), penalty function (Epenoy, 2011), conic optimization (Lu and Liu, 2013), nonlinear optimal controller (Ma et al., 2007), Gauss pseudospectral method (Boyarko et al., 2011), and model predictive control (MPC) (Wang, 2009). Among them, the MPC, based on the receding horizon control strategy, appeals because of its ability to update the state variables

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online to satisfy multiple and changing operational constraints. The latter are resulted from the continuous update of states of non-cooperative targets from observation (Breger and How, 2008, 2007; Gavilan et al., 2012; Cairano et al., 2012; Hartley et al., 2012; Hartley and Maciejowski, 2014; Leomanni et al., 2014a,b). Although effective, the time-varying control output generated by existing MPC control algorithms is not viable for most existing thrusters that work in an on–off mode with constant magnitudes (Lian and Tang, 2013). The situation is further complicated by the delay in the on–off switch. Modulating continuous control into equivalent and discrete on–off pulses poses a challenging task for spacecraft designers (Leomanni et al., 2014a,b; Vazquez et al., 2015). Many on–off modulation algorithms have been proposed in the literature, such as, the Schimitt trigger control, the pseudo rate modulator, the integrated pulse frequency modulator and the pulse-width pulse-frequency (PWPF) modulator (Song et al., 1999; Hu and Ma, 2005). Compared with others, the PWPF is widely used in spacecraft attitude control systems due to its advantages in controlling the on–off switching-states of thrusters in terms of closed-to-linear operation, reduced propellant consumption, high accuracy and adjustability to advanced control algorithms (Song and Agrawal, 2001; Hu and Ma, 2005). Up to date, few has attempted to integrate the PWPF modulation with the state dependent modal predictive control (SDMPC) in ARPO.

In this paper, a new scheme of PWPF based SDMPC is proposed in ARPO using constant-magnitude thrust on–off control. Practical constraints on thrusters' magnitude and Light-of-Sight (LOS) are imposed in the SDMPC algorithm to prevent the collision between two spacecraft. The control objective is to optimize the control accuracy of states and control smoothness subjected to the discontinuous on–off propulsion systems. The resulting optimal control problem is formulated into a series of constrained quadratic programming problems and solved at sampling instants. The optimized continuous control input at each sampling instant is converted to a sequence of on–off pulses by a PWPF modulator to control thrusters' firing frequency and duration. Subsequently in the numerical simulation, the effectiveness and accuracy of the newly proposed integration of SDMPC and PWPF modulation is demonstrated by comparing with its continuous control counterpart. The accuracy of residual relative positions and velocities of the chaser at the rendezvous are almost the same as that obtained by the scheme of continuous control force. The new approach is more energy efficient. Finally, the current work focuses on the viability of the proposed integration of SDMPC and PWPF modulation in the orbital rendezvous. For the sake of simplicity, the relative attitude tracking and control of the chaser is not considered. This important part in a practical rendezvous and docking mission will be included in the future work.

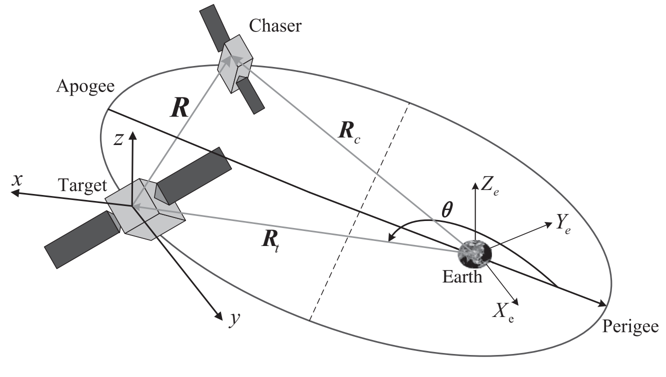

Fig. 1. Schematic of spacecraft rendezvous and related coordinate systems.

2. Dynamic formulation of spacecraft rendezvous

Consider a typical autonomous rendezvous and docking maneuver where a chaser approaches a target in a central gravitational field of the Earth. The spacecraft are described in an inertial frame $OX_eY_eZ_e$ centered at the Earth as shown in Fig. 1. The center of mass (CM) of the target is assumed moving in an elliptic orbit around the Earth. The orbital radii of CM of the target and the chaser are denoted by $\boldsymbol{R_t}$ and $\boldsymbol{R_c}$ in the inertial frame, respectively. Then, the relative distance between the CMs of two spacecraft is determined by $R = \|\boldsymbol{R}_c - \boldsymbol{R}_t\| = \sqrt{x^2 + y^2 + z^2}$. The relative motion of the chaser with respect to the target can be described in a LVLH (Local Vertical/Local Horizontal) coordinate system (o-xyz) that is centered at the CM of the target as shown in Fig. 1. The x-axis is aligned with $\boldsymbol{R_t}$ pointing outwards, the z-axis is aligned with the vector product of $\boldsymbol{R_t}$ and instantaneous orbital velocity of the target, and the y-axis completes a right-handed coordinate system.

If the distance between two spacecraft is sufficiently small compared to the orbit radius but sufficiently large compared to the largest dimension of spacecraft, the relative motion can be described by the linearized Tschauner–Hempel (TH) equation in the neighborhood of the target orbiting in an elliptic orbit (Inalhan et al., 2002; Van der Ha and Mugellesi, 2002; Vaddi et al., 2003), such that,

$$\begin{Bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{Bmatrix} = -2\begin{bmatrix}0 & -\dot{\theta} & 0\\ \dot{\theta} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{Bmatrix} - \begin{bmatrix}-\dot{\theta}^2 & 0 & 0\\ 0 & -\dot{\theta}^2 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}x\\ y\\ z\end{Bmatrix} - \begin{bmatrix}0 & -\ddot{\theta} & 0\\ \ddot{\theta} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}x\\ y\\ z\end{Bmatrix} + \frac{\mu}{R_t^3}\begin{Bmatrix}2x\\ -y\\ -z\end{Bmatrix} + \frac{1}{m_c}\begin{Bmatrix}F_x\\ F_y\\ F_z\end{Bmatrix} \tag{1}$$