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Extreme values of relative distances for spacecraft in elliptic displaced orbits

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Abstract

This paper provides a framework to obtain a semi-analytical approximation of extreme values of relative distances between two spacecraft that cover elliptic displaced orbits. The relative motion is described in the rotating reference frame of the chief spacecraft and is parameterized with a new set of displaced orbital elements. The extreme values of the radial, along-track and cross-track distance are analytically evaluated (as roots of suitable algebraic equations) both for quasi-periodic orbits in the incommensurable case, and for periodic orbits in the 1:1 commensurable case. In particular, in the 1:1 commensurable case a Fourier series expansion is used to obtain a time-explicit expression of the relative motion. Finally, some illustrative examples are presented to validate the correctness of the proposed method.

Keywords: Elliptic displaced orbits; Relative motion bounds; Formation flying

1. Introduction

The development of advanced materials, such as the graphene (Matloff, 2012), and the practical use of innovative propulsion system concepts, such as the photonic solar sail (Tsuda et al., 2011a,b; Mori et al., 2010; Johnson et al., 2011a,b, 2012), have contributed to a growing interest towards non-Keplerian orbits (McKay et al., 2011; Mengali and Quarta, 2009), due to their potential benefits offered to astronomical missions. In particular, artificial Lagrange orbits in a three-body dynamical system (McInnes et al., 1994; Baoyin and McInnes, 2005; Baoyin and McInnes, 2006a,b) are capable of monitoring solar plasma storms (Prado et al., 1996), while displaced orbits in two-body problems (McInnes, 1997; McInnes and Simmons, 1992a; McInnes and Simmons, 1992b) can be used as planet pole sitters (Ceriotti et al., 2014). In principle, a displaced orbit, that is, an orbit whose orbital plane does not contain the primary's center-of-mass, can be generated either by means of photonic solar sails (McInnes, 1998; Gong et al., 2008b), or by the more recent electric solar wind sails (Janhunen, 2004; Mengali and Quarta, 2009), whereas more conventional propulsion systems are known to be inadequate for these applications (McInnes et al., 1999).

In this context, most of the available literature is dedicated to the study of circular displaced orbits (McInnes, 1997; Ceriotti et al., 2014; Gong et al., 2008b; Mengali and Quarta, 2009) or to artificial Lagrange points (McInnes et al., 1994; Baoyin and McInnes, 2005; Baoyin and McInnes, 2006a,b) in the restricted three-body problem, whereas elliptic displaced orbits have attracted smaller interest. However, some celestial bodies such as Mercury

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Nomenclature

a	semimajor axis of displaced orbit, [au]
b	semiminor axis of displaced orbit, [au]
E	eccentric anomaly, [rad]
f	true anomaly, [rad]
H	orbit displacement, [au]
$\hat{\boldsymbol{k}}$	unit vector of z-axis
i	inclination of displaced orbit, [rad]
$\hat{\boldsymbol{i}}$	unit vector of x-axis
J_h	Bessel function of the first kind of order h
$\hat{\boldsymbol{j}}$	unit vector of y-axis
M	mean anomaly, [rad]
n	mean motion, [rad/day]
O	Sun's center-of-mass
o	focus of displaced orbit
$\mathfrak{R}$	manifold
$\boldsymbol{r}$	spacecraft position vector in $\mathcal{T}_I$ (with $r = \lVert\boldsymbol{r}\rVert$), [au]
S	spacecraft center-of-mass
t	time, [days]
$\mathcal{T}_I$	inertial reference frame
$\mathcal{T}_P$	perifocal reference frame
$\mathcal{T}_R$	rotating reference frame
$\mathbb{T}_{PI}$	transformation matrix between $\mathcal{T}_P$ and $\mathcal{T}_I$
$\mathbb{T}_{PR}$	transformation matrix between $\mathcal{T}_P$ and $\mathcal{T}_R$
$\mathbb{T}_{P_DP_C}$	transformation matrix between $\mathcal{T}_{P_D}$ and $\mathcal{T}_{P_C}$
T_{ij}	(i,j) entry of matrix $\mathbb{T}_{P_DP_C}$
ρ_x, ρ_y, ρ_z	components of relative position vector in the chief's rotating frame, [au]
$\boldsymbol{\rho}$	relative position vector, [au]
Ω	right ascension of the ascending node of displaced orbit, [rad]
ω	argument of periapsis of displaced orbit, [rad]

Subscripts

C	chief
D	deputy
n	numerical solution

Superscripts

T	transpose
★	extreme value
∧	unit vector

(or some near-Earth asteroids) track orbits with considerable eccentricity and, in that case, a full-time observation of their pole region could not be obtained using circular displaced orbits. An interesting option for those mission scenario is the use of elliptic displaced orbits. In particular, Gong and Li, 2014 point out that a photonic solar sail with reflection control devices (electrochromic panels) could be able to achieve and maintain such kind of non-Keplerian orbit. This is possible by exploiting the fact that the mean optical properties of the sail's film can be adjusted, within some limits, by switching on or off the state of each electrochromic panel (Gong and Li, 2014; Mu et al., 2015a,b; Hu et al., 2016).

Moreover, scientific explorations of some celestial bodies that cover eccentric orbits, require multi-aspect observations that can only be accomplished using a formation of spacecraft. For instance, the interaction of solar wind with the magnetic field of Mercury is still, to some extent, an unknown phenomenon. An exhaustive comprehension of the magnetotail structure and dynamics of Mercury is possible only if a correlation between these two acting causes may be found (Aliasi et al., 2015), which requires the simultaneous observation and measurement of both the magnetotail and the solar wind from different positions using multiple spacecraft. However, the only existing papers on solar sail relative motion around displaced orbits use a linearized mathematical model (Gong et al., 2007a, 2008a, 2007b; Gong et al., 2011), and thus can only be applied to small-distance and short-term missions. Especially for a large baseline formation flying, a nonlinear insight is indispensable for looking at some inherent properties of the formation structure, such as the bounds and the geometric topology of relative motion (Gurfil and Kholshevnikov, 2006), which might be useful for collision avoidance and communication analysis.

The aim of this paper is to give a comprehensive analysis of closed-form solutions and bounds of relative motion between elliptic displaced orbits, thus generalizing the recently developed theory (Wang et al., in press) wherein the relative motion between circular displaced orbits has been investigated. A new set of displaced orbital elements is first introduced to describe an elliptic displaced orbit and to define its orientation with respect to an inertial reference frame. Based on the newly defined parameters, a closed-form solution expressed in the chief sail's rotating frame is derived without the need of solving or linearizing the relative dynamics. The relative motion emanated from the parametric solution is quasi-periodic for the incommensurable case and periodic for the commensurable case. In the commensurable case, the ratio between the mean motion of the two elliptic displaced orbits is a rational number, whereas for the incommensurable case the ratio is an irrational number. In both cases, the relative motion evolves along its invariant manifold that presents a well-defined accessible domain (Topputo, 2016; Mingotti et al., 2012).

The general solution is then used to look for the bounds along each coordinate axis assuming a small value of orbital eccentricity. For the 1:1 commensurable case, a first-order closed-form approximate solution is obtained, and