Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 57 (2016) 1641–1651

ADVANCES IN SPACE RESEARCH (a COSPAR publication)

www.elsevier.com/locate/asr

Hybrid perturbation methods based on statistical time series models

Juan Félix San-Juan [a,*], Montserrat San-Martín [a], Iván Pérez [a], Rosario López [b]

[a] Scientific Computing Group (GRUCACI), University of La Rioja, 26004 Logroño, Spain
[b] Scientific Computing Group (GRUCACI), Center for Biomedical Research of La Rioja (CIBIR), 26006 Logroño, Spain

Received 30 October 2014; received in revised form 9 May 2015; accepted 16 May 2015
Available online 22 May 2015

Abstract

In this work we present a new methodology for orbit propagation, the hybrid perturbation theory, based on the combination of an integration method and a prediction technique. The former, which can be a numerical, analytical or semianalytical theory, generates an initial approximation that contains some inaccuracies derived from the fact that, in order to simplify the expressions and subsequent computations, not all the involved forces are taken into account and only low-order terms are considered, not to mention the fact that mathematical models of perturbations not always reproduce physical phenomena with absolute precision. The prediction technique, which can be based on either statistical time series models or computational intelligence methods, is aimed at modelling and reproducing missing dynamics in the previously integrated approximation. This combination results in the precision improvement of conventional numerical, analytical and semianalytical theories for determining the position and velocity of any artificial satellite or space debris object. In order to validate this methodology, we present a family of three hybrid orbit propagators formed by the combination of three different orders of approximation of an analytical theory and a statistical time series model, and analyse their capability to process the effect produced by the flattening of the Earth. The three considered analytical components are the integration of the Kepler problem, a first-order and a second-order analytical theories, whereas the prediction technique is the same in the three cases, namely an additive Holt–Winters method.

Keywords: Artificial satellite theory; Orbit propagator; Hybrid perturbation method; Time series

1. Introduction

The equations of the perturbed motion of a satellite can be written as a set of 3 second-order or 6 first-order ordinary differential equations. The orbit propagation problem consists in computing the position and velocity of the satellite at a given final time t_f, from the position and velocity at a given initial time t_1. Classically, the techniques used to solve this problem have been three.

The first two methods are known as general and special perturbation techniques. General perturbation techniques are based on the analytical integration of the satellite's equations of motion using perturbation theories (Deprit, 1969; Giacaglia, 1964; Hori, 1966, 1971; Krylov and Bogoliubov, 1947; Morrison, 1965). These techniques provide approximate analytical solutions (Aksnes, 1970; Brouwer, 1959; Hoots and Roehrich, 1980; Hoots and France, 1987; Kinoshita, 1977; Kozai, 1962; Lyddane, 1963) valid for any set of initial conditions. These solutions are explicit functions of time, physical parameters and integration constants, which are mainly characterised by retaining the essential behaviour of the motion. It is worth noting that most analytical theories currently in use only consider very basic models of external forces, because in

* Corresponding author. Tel.: +34 941299440; fax: +34 941299460.
E-mail addresses: juanfelix.sanjuan@unirioja.es (J.F. San-Juan), montse.sanmartin@unirioja.es (M. San-Martín), ivan.perez@unirioja.es (I. Pérez), rlgomez@riojasalud.es (R. López).

http://dx.doi.org/10.1016/j.asr.2015.05.025
0273-1177/

some cases their corresponding analytical expressions can be too cumbersome. Furthermore, only low-order approximations are taken into account because analytical expansions for the higher-order solutions may become unmanageably long. Some of these theories can even implement truncated dynamic parameter expansions, so that their accuracy and computational efficiency are closely related to the initial assumptions.

On the other hand, special perturbation methods (Berry and Healy, 2004; Kinoshita and Nakai, 1989; Long et al., 1989) refer to the accurate numerical integration of the equations of motion, including any external forces, even those in which analytical manipulations are complicated, which makes it necessary to use small steps in order to integrate the equations of motion. General perturbation methods produce more computationally efficient propagators although are not as accurate as those developed using special perturbation techniques.

Finally, the third approach is the semianalytical technique (Cefola et al., 2009; Liu and Alford, 1980; Neelon et al., 1997), which combines and takes advantage of the best characteristics of both the general and special perturbation techniques. This approach allows to include any external forces in the equations of motion, which are simplified using analytical techniques. Thus, the transformed equations of motion can be integrated numerically in a more efficient way by using longer integration steps.

Current needs for Space Situational Awareness require improving orbit propagation of space objects in different ways, including the efficient short-term propagation of catalogues of thousands of objects, the accurate very-long-term propagation needed for designing disposal strategies, the instant propagation of fragmentation models, or the propagation of uncertainties in observed orbits of *potentially hazardous objects*, among others.

Improvement in the models to be integrated constitutes a basic line of research, together with the use of advanced computer architectures based on parallel processing. Additional improvement can be achieved by combining both integrating and forecasting techniques, which we have called hybrid methods.

In this work we present the hybrid perturbation theory, which may combine any kind of the aforementioned integration techniques with forecasting techniques based on statistical time series models (Chan, 2010; Trapletti and Hornik, 2011; San-Juan et al., 2012) or computational intelligence methods (Pérez et al., 2013). This combination allows for an increase in the accuracy of the numerical, analytical or semianalytical theories for predicting the position and velocity of any artificial satellite or space debris object, through the modelling of higher-order terms and other external forces not considered in those initial theories, as well as some physical effects not accurately modelled by the mathematical equations. The final goal of hybrid methodology is to complement the mathematical model of an orbiter dynamics, which is never a completely faithful representation of physical phenomena, with real dynamics provided by real observations, thus yielding a more accurate representation of real behaviour. As a first step in the process to eventually include unmodelled physical effects in the formulation of the problem, we start by considering a basic perturbation, J_2, and check the capability of the hybrid propagator to grasp its dynamics. In this process we simulate real observations by means of numerically generated ephemeris through an 8th order Runge–Kutta method (Dormand and Prince, 1989).

The aim of this paper is to develop a family of hybrid orbit propagators based on three different orders of approximation of an analytical theory, in order to model the effect produced by the flattening of the Earth so that this technique can be validated. These hybrid orbit propagators incorporate the integration of the Kepler problem in the first case, a first-order analytical theory in the second case and a second-order analytical theory in the last case as the integration techniques; the forecasting technique is an additive Holt–Winters method in the three cases.

This paper is organised as follows. Section 2 describes the concept that underlies hybrid perturbation methodology. Section 3 outlines the second-order analytical theory PPD that, together with its first-order and zero-order approximations, constitutes the base for the three hybrid propagators to be developed in the following sections. Section 4 describes the Holt–Winters method, an exponential smoothing technique used in this paper as the forecasting part of the hybrid propagators. In Section 5, the construction of the three hybrid propagators is detailed, paying special attention to the preliminary statistical analysis of control data, which is important in order to choose the most appropriate sampling rate for the time series to be processed. Results are analysed, and compared to the conventional analytical propagation results, for a set of 9 LEO satellites. Finally, Section 6 summarises the study and remarks some interesting findings.

2. Hybrid perturbation methodology

A hybrid perturbation theory is a methodology for determining an estimation of the position and velocity of any orbiter, which may be an artificial satellite or space debris object, at a final instant t_f, in some set of canonical or non-canonical variables, $\hat{\boldsymbol{x}}_{t_f}$.

In a first phase, an integration method $\mathcal{I}$ is needed in order to calculate a first approximation, $\boldsymbol{x}^{\mathcal{I}}_{t_f}$, from the position and velocity at an initial instant $t_1, \boldsymbol{x}_{t_1}$:

$$\boldsymbol{x}^{\mathcal{I}}_{t_f} = \mathcal{I}(t_f, \boldsymbol{x}_{t_1}). \tag{1}$$

This approximation can include some inaccuracies derived from the facts that, for the sake of manageability of the resulting expressions and affordability of the subsequent computations, not all the external forces are usually taken into account in the physical model, and only low-order approximations are considered. Additional