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Relative motion and visibility evaluation in GNSS constellation: A projection method

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Abstract

Using crosslinks to network the satellites in a GNSS constellation has been regarded as one of the most important trends for the GNSS modernisation. The motion and visibility of the satellites w.r.t. each other in the constellation have to be evaluated as an prerequisite to many tasks, especially to the inter-satellite network design, measurement scheduling and pointing-related issues. Thus far, very few analytical studies have been done on this issue for GNSS constellations. In this paper, a projection method is proposed to evaluate the relative motion and visibility. By projecting the relative trajectory orthogonally onto the orbital plane of the base satellite, it is much easier to study the geometric characteristics of the relative motion and the effect of constellation parameters and field-of-view constraint on the relative visibility. Given that GNSS constellations are designed based on Walker constellation, the relative motion pattern is explored algorithmically with index mapping techniques and the always visible relationship is visually modelled using a graph which in most cases shows a symmetry structure when the vertices are placed in a proper order. As demonstration of our method, a thorough experimental validation on the nominal GPS, GLONASS, Galileo and BeiDou-M constellation is proposed. The result explicitly reveals that the relative motion and visibility varies according to the parameters of the constellations.

Keywords: GNSS; Crosslink; Relative motion; Relative visibility; Always visibility graph

1. Introduction

Crosslinks are the wireless links for performing relative pseudo-range measurement and data exchange among a set of satellites. Enhanced by crosslinks, the navigation satellites in a GNSS (Global Navigation Satellite System) constellation can be networked to support on-board orbit and clock determination processing and prediction, so as to be enabled to provide PNT (Positioning, Navigation and Timing) services without contacting the ground infrastructure for long periods of time (a.k.a. autonomous navigation) (Ananda et al., 1990; Abusali, 1998; Maine et al., 2003). Besides, with better satellite-to-satellite observations and data exchanging, the orbit and clock prediction accuracy and system operation efficiency can be improved (Maine et al., 2004; Luba et al., 2005; Marquis and Shaw, 2011). Therefore, using crosslinks to network the navigation satellites has been regarded as one of the most significant directions in the course of the GNSS modernisation (Amarillo, 2011; Xu et al., 2012).

As a prerequisite to many tasks such as the crosslink antennas design and control, network design and analysis, interference assessment, satellite attitude error budgets and other pointing-related issues with the GNSS constellation, the motion and visibility of the satellites relative to each other have to be evaluated effectively. Although there have been sophisticated analytical studies for the small-scale relative motion (for which the orbits of two spacecrafts are very close to each other), for instance, the

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Hill–Clohessy–Wiltshire equations have been extensively used in the field of spacecraft formations and rendezvous (Clohessy and Wiltshire, 1960), the small relative distance assumption and relevant approximation would be no longer valid in GNSS constellations. Thus far, the analytical studies on the large-scale satellites relative motion and visibility are limited and insufficient, especially for the GNSS constellations. Most of the current studies are based on the numerical simulation performed on computers, which however, require substantial efforts to provide the physical insight or broad overview critical for conceptual mission design, constellation design and analysis, or the analysis of alternative approaches (Wertz, 2009).

For the best compromise between global continuous coverage, stable broadcast navigation signal strength, acceptable numbers of satellites and cost, all the current primary GNSS constellations are designed based or derived from the Walker constellation that can be described by the notation $N/P/F : i, h$ (Kaplan and Hegarty, 2005). For instance, the notation for the medium Earth orbit (MEO) subsystem of BeiDou constellation is $24/3/1 : 55^\circ, 21528$ km. More specifically, a Walker constellation consists of a total of N satellites in circular orbits of altitude h and inclination i, where the P orbital planes are equally spaced around the equatorial plane and $S = N/P$ satellites are equally spaced within each orbital plane. The offset in argument of latitude between the first satellites in each adjacent orbital planes is determined by $F \times 2\pi/N$, where F is an integer with $0 < F < P$. Generally, h is around 20000 km to achieve a balance between the requirements of the navigation signal transmission power and ground coverage, which could also benefit the satellite relative visibility as the angle subtended by the entire Earth disk decreases with the increase of satellite altitude.

It is assumed that the crosslink antennas are microwave antennas with axes pointing to the Earth and mounted on the satellite's Nadir surface. When two satellites are moving outside of the coverage of their crosslink antennas or when they are moving into the shadow of the Earth and its atmosphere, they are invisible to each other and the crosslink measurements between them is disrupted. Subsequently, we only consider satellites within the field-of-view (FOV) mask of Nadir-looking crosslink antennas, i.e., with a proper elevation angle of the Line-Of-Sight (LOS) vector above the satellite's Z-plane. The FOV mask is determined by two angles: ϑ_{min} and ϑ_{max}, where ϑ_{max} is defined by the Earth radius R_E, the atmosphere thickness h_a and the satellite altitude h such that $\vartheta_{max} = \arccos\left(\frac{R_E+h_a}{R_E+h}\right)$; ϑ_{min} is calculated from the crosslink antenna's half beam width (for wide-beam antennas) or scanning off-axis angle (for narrow-beam antennas) ζ such that $\vartheta_{min} = \pi/2 - \zeta$. Meanwhile, because all the satellites in a GNSS constellation are assumed to have same attitude, it is implied that if satellite A observes satellite B under a certain elevation ϑ, satellite B observes satellite A under the same elevation. Without any ambiguity, these two equivalent elevation angles are simply described as the relative elevation of two satellites. Two satellites are mutually visible only when their relative elevation ϑ lies within the FOV mask, i.e. $\vartheta_{min} \leqslant \vartheta \leqslant \vartheta_{max}$, otherwise they are mutually invisible.

The rest of this paper is organised as follows: the equations and geometry of the relative motion in a GNSS constellation is analysed in Section 2. In Section 3, the relative visibility is evaluated under the constraint of given FOV mask, including the determination process and representation. In Section 4, a thorough experimental evaluation on GPS, GLONASS, Galileo and BeiDou constellation is provided with the same antenna capabilities, as a demonstration of the projection method.

2. Relative motion in GNSS constellation

In this section, we begin by introducing the coordinate frames for the analysis on the motion of the satellites w.r.t. an individual satellite in a GNSS constellation. Afterwards, we will discuss the relative motion of two satellites in circular orbits so as to study the global properties, including the geometry, projection and distribution of the relative trajectories. The GNSS constellation is assumed to be a Walker constellation and that all satellites are travelling in circular orbits with the same radii and in orbits with the same inclination.

2.1. Coordinate frame definition

It is convenient to use an Earth-centred inertial (ECI) coordinate system for discussing the orbital motion of the satellites w.r.t. the Earth. In typical ECI coordinate systems, the origin is located at the Earth's centre of mass, the xy-plane is the equatorial plane, and the orientation of the frame is fixed with respect to the celestial sphere. A commonly used ECI system for GNSS is defined with the equatorial and ecliptic planes at 12 h UTC on January 1, 2000, named J2000 system and denoted as $\Re(O_E; X, Y, Z)$. As shown in Fig. 1a, its first axis O_EX is permanently fixed in the direction from the Earth's centre of mass to the vernal equinox, the third axis O_EZ is normal to the equatorial plane in the direction of the North pole, and the second axis O_EY is chosen so as to complete a right-handed coordinate system, all at the aforementioned epoch. For a satellite in a circular orbit, its coordinates at epoch t are governed by four elements, e.g. (a, Ω, i, u_0), where a is the semi-major axis, Ω is the right ascension of the ascending node, i is the inclination w.r.t. the equatorial plane, and u_0 is the argument of latitude at an initial epoch t_0.

To figure out the other satellites' apparent motion as observed from an arbitrarily chosen base satellite in the constellation, we use a satellite-centred, Earth-pointing, orbital coordinate system co-rotating with the base satellite (hereafter referred to as a rotated $\mathcal{R}$-system and denoted as $\Re(O_s; x, y, z)$). As illustrated in Fig. 1a, the origin O_s of a rotated $\mathcal{R}$-system is fixed on the centre of the base satellite