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Analytical solutions to optimal underactuated spacecraft formation reconfiguration

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Abstract

Underactuated systems can generally be defined as systems with fewer number of control inputs than that of the degrees of freedom to be controlled. In this paper, analytical solutions to optimal underactuated spacecraft formation reconfiguration without either the radial or the in-track control are derived. By using a linear dynamical model of underactuated spacecraft formation in circular orbits, controllability analysis is conducted for either underactuated case. Indirect optimization methods based on the minimum principle are then introduced to generate analytical solutions to optimal open-loop underactuated reconfiguration problems. Both fixed and free final conditions constraints are considered for either underactuated case and comparisons between these two final conditions indicate that the optimal control strategies with free final conditions require less control efforts than those with the fixed ones. Meanwhile, closed-loop adaptive sliding mode controllers for both underactuated cases are designed to guarantee optimal trajectory tracking in the presence of unmatched external perturbations, linearization errors, and system uncertainties. The adaptation laws are designed via a Lyapunov-based method to ensure the overall stability of the closed-loop system. The explicit expressions of the terminal convergent regions of each system states have also been obtained. Numerical simulations demonstrate the validity and feasibility of the proposed open-loop and closed-loop control schemes for optimal underactuated spacecraft formation reconfiguration in circular orbits.

Keywords: Indirect optimization; Spacecraft formation flying; Formation reconfiguration; Underactuated control; Adaptive sliding mode control

1. Introduction

Spacecraft formation flying (SFF) that consists of a cluster of spacecraft has been identified as a key enabling technology for future space missions due to the advantages over a traditional monolithic spacecraft such as decreased cost and risk, enhanced reliability and survivability (Huang et al., 2014a). It also offers increased flexibility as a result of the capability of formation reconfiguration that the spacecraft reposition themselves with respect to each other to adapt to different space missions (Sabol et al., 2001). As a major issue in SFF technology, optimal spacecraft formation reconfiguration has received wide research interests. Vaddi et al. (2005) designed optimal formation establishment and reconfiguration methods using impulsive control. Optimal strategies for linearized balanced-energy formation flying maneuvers using continuous thrusts have been proposed by Rahmani et al. (2006) via an indirect optimization method. Lee and Park (2011) derived approximate analytical solutions to optimal reconfiguration problems in linearized perturbed orbits via the indirect method, too.

Trajectory optimization methods are usually grouped into two categories, namely, indirect and direct methods (Conway, 2012). In indirect methods, the necessary

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optimality conditions derived from the minimum principle are solved to generate exact solutions to the trajectory optimization problems, which are thus inadequate for complex problems that are hard to derive analytical solutions. For complex nonlinear trajectory optimization problems, the direct methods are more generally used, which transcribe the optimal control problem into a nonlinear programming (NLP) and solve the resulting NLP by numerical methods. Several direct methods have been used to solve optimal formation reconfiguration problems. Wu et al. (2009, 2011) investigated nonlinear optimization problems of formation reconfiguration by Legendre pseudospectral method that belongs to direct methods. Huntington and Rao (2008) addressed the problem of how to reconfigure a tetrahedral formation in a fuel-optimal way using the Gauss pseudospectral method. Similar direct methods have also been utilized to achieve optimal reconfiguration propelled by the geomagnetic Lorentz forces or the inter-spacecraft Coulomb forces (Huang et al., 2014a, 2015a; Inampudi and Schaub, 2012).

In most of the previous works that handled the optimal reconfiguration problems propelled by continuous thrusts, the dynamical system of SFF is assumed to be fully actuated that the number of the degrees of freedom (DOFs) of the system is equal to the number of the dimension of independent control inputs. In other words, an independent thruster exists in each of the radial, in-track, or normal direction. However, when a certain thruster breaks down, the dynamical system of SFF would turn into an underactuated one. Generally, underactuated systems refer to the systems in which the dimension of the configuration space exceeds that of the control input space, that is, the number of the control inputs is less than the DOFs to be controlled (Xin and Liu, 2014). Obviously, the previous fully-actuated SFF control schemes could not accommodate the underactuated cases and it may lead to the mission failure. Therefore, it is necessary to design underactuated controllers to guarantee formation reconfiguration with the loss of thrust in a certain direction. Compared to the fully-actuated reconfiguration problems, fewer works deal with the underactuated ones. Leonard et al. (1989) firstly examined the feasibility of formation keeping by using the differential drag between spacecraft that acts in the in-track direction only. Kumar et al. (2007) designed a linear controller for SFF that uses the in-track thrust only. Similar problem has also been investigated by Kumar et al. (2011). Varma and Kumar (2012) proposed another linear sliding mode controller (SMC) for SFF using the in-track differential aerodynamic drag. Different from aforementioned underactuated controllers with the loss of radial thrust, Godard et al. (2014) designed SMCs for underactuated SFF with the loss of either the radial or the in-track thrust. As can be seen, current works mainly concentrate on the feasibility of underactuated formation reconfiguration in the absence of radial or in-track control, but seldom solve optimal control problems of underactuated reconfiguration. Since it is always desirable to perform orbital maneuvers in a fuel-optimal or energy-optimal way, optimal underactuated controllers for reconfiguration are valuable to be designed. Furthermore, considering that concise results could be obtained by using the linearized dynamical system of SFF in circular orbits, the indirect optimization method is adopted in this paper to derive precise analytical solutions to underactuated formation reconfiguration.

Firstly, a linearized dynamical model of underactuated SFF is introduced to conduct controllability analysis for either underactuated case. Then, based on the controllability analysis, indirect methods are used to derive analytical solutions to underactuated formation reconfiguration. In generating the optimal trajectories of reconfiguration for either underactuated case, two kinds of final conditions constraints are considered, namely, the fixed and free final conditions. For the fixed final conditions, the terminal relative state vector (i.e., final conditions constraints) has been predetermined as a given one that satisfies the geometry constraints for the final desired formation. However, for the free final conditions, the terminal relative states are not determined in advance but treated as optimization variables subject to the geometry constraints of the final formation. In other words, the terminal relative state vector is not a single fixed one but could be any feasible one as long as the terminal geometry constraints are satisfied. Comparisons are thus then made between the results obtained with these two kinds of final constraints. Furthermore, considering the external disturbances, linearization errors, and system uncertainties that may drift the desired trajectory of reconfiguration, closed-loop adaptive SMCs (ASMCs) have also been proposed to ensure optimal trajectory tracking for both underactuated cases.

The organization of this paper proceeds as follows. The dynamical model of underactuated SFF and the corresponding controllability analysis are presented in Section 2. Sections 3 and 4 introduce the indirect methods in generating the open-loop optimal solutions to underactuated reconfiguration with fixed and free final conditions, respectively, followed by the closed-loop underactuated controllers designed in Section 5. Theoretical results are verified by the numerical simulations shown in Section 6, and Section 7 concludes the paper.

2. Dynamical model of underactuated SFF

Consider a chief spacecraft in an Earth orbit and a deputy spacecraft flying nearby, which constitutes a formation together with the chief. As shown in Fig. 1, $O_E X_I Y_I Z_I$ is an Earth-centered inertial frame with O_E being the center of Earth. $O_C xyz$ is a local vertical local horizontal (LVLH) frame with its origin located at the center-of-mass (c.m.) of the chief, O_C, where x axis is along the radial direction, z axis is aligned with the normal direction of the chief's orbital plane, and y axis completes the right-handed Cartesian frame. O_D is the c.m. of the deputy. The position and velocity vectors of the deputy with respect to the chief