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Gaussian Analytic Centroiding method of star image of star tracker

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Abstract

The energy distribution of an actual star image coincides with the Gaussian law statistically in most cases, so the optimized processing algorithm about star image centroiding should be constructed also by following Gaussian law. For a star image spot covering a certain number of pixels, the marginal distribution of the gray accumulation on rows and columns are shown and analyzed, based on which the formulas of Gaussian Analytic Centroiding method (GAC) are deduced, and the robustness is also promoted due to the inherited filtering effect of gray accumulation. Ideal reference star images are simulated by the PSF (point spread function) with integral form. Precision and speed tests for the Gaussian Analytic formulas are conducted under three scenarios of Gaussian radius (0.5, 0.671, 0.8 pixel), The simulation results show that the precision of GAC method is better than that of the other given algorithms when the Gaussian radius is not bigger than 5×5 pixel window, a widely used parameter. Above all, the algorithm which consumes the least time is still the novel GAC method. GAC method helps to promote the comprehensive performance in the attitude determination of a star tracker.

Keywords: Gaussian distribution; Star tracker; PSF; Analytic formula; Centroiding

1. Introduction

In most cases, the energy distribution of an image spot of star tracker accords approximately with the two-dimensional Gaussian distribution. To make the best use of some image processing algorithms, a kind of de-focus technology, namely not focusing intentionally, is usually adopted to expand the area of the image spot in optics adjustment (Liebe, 2002). In addition, there is inherent optics aberration, so the combination of the above two factors causes the star image spot to cover a certain number of pixels, which helps to use image edge detection algorithms to obtain better centroiding precision of sub-pixel level (Wang et al., 2009).

There exist a variety of classical centroiding methods, among which the most widely-used and robust is the method of "Center Of Gravity" (Tjorven et al., 2013), in reference Shortis et al. (1994) called "gray scale centroid", another is squared gray scale centroid method (Shortis et al., 1994), the above mentioned two are virtually of the basic idea of primary and secondary moment with regard to gray or squared gray and distance, or as can be also thought of as that the gray or squared gray serves as the weight of the distance from the centroid, the above two moment methods feature the simplicity of calculation and high speed, but with the poorest accuracy (Tjorven et al., 2013). Usually, the error curve of the moment method is of periodic form, and needs to be compensated to increase the precision (Giancarlo and Domenico, 2003; Jia et al., 2010).

Another class of centroiding method is by estimation and fitting, among which the most typical are 1D and 2D Gaussian Least Squares Fit Method and Gaussian Grid Algorithm (GGA), the former two are of the highest accuracy but suffer from the most intensive computation, while

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the latter has less computation and the second highest accuracy behind the 1D and 2D Gaussian Least Squares Fit method (Tjorven et al., 2013), an acceptable comprehensive performance. And yet the attribute of time saving reported by Tjorven is found not as true as what was proven in actual tests shown in Table 2.

As for GGA, its deduction involves several approximate treatments including the LSM estimation based on cost function, numeric fitting and first order Taylor expanding and cutting, so in a strict sense this method should not belong to the analytical category even though with final explicit expressions. What is more, the computation of weights w_{ij}, 6 times of natural logarithm, as well as the great deal of multiplication about coefficients A_{ij} and B_{ij} equations listed in Appendix A (Tjorven et al., 2013), will bring about a high cost of time for GGA.

In this paper, a novel method is constructed through a strict analytical routine, called Gaussian Analytic Centroiding (GAC) method. Since a defocused spot imaged by parallel light follows a 2D Gaussian energy distribution approximately for most lenses, the optimized processing algorithm about star image centroiding should be constructed also by following Gaussian law. What GAC is different from 1D or 2D Gaussian Least Squares method lies in that no LSM estimation or fitting process is involved in the its deduction. GAC also has the same merits as GGA has, such as no requirement of initial estimate of the amplitude and the Gaussian parameter σ_x and σ_y (Tjorven et al., 2013). The following sections will describe its deduction and verification process of GAC.

2. Formula of Gaussian Analytic Centroiding method

The mean-square error σ_r of the 2D Gaussian energy distribution of an image spot is defined as Gaussian radius here, representing for the size of the image spot, to comprehensively embody the de-focus level and various optics aberrations, the area of $3\sigma_r$ circle collects about 99.73% energy of the image spot, as shown in Fig. 1.

The value of σ_r is not only a reference variable in focus adjustment, but also an important coefficient used in centroiding algorithm, which will affect the accuracy of centroiding, the success rate of star map recognition and even the final precision of attitude determination. A 2D Gaussian energy distribution contour of an image spot is shown above.

In the below Fig. 2, the dashed $3\sigma_r$ bigger circle area is the coverage of the image spot by de-focused lens, involving 4×4 pixels, where its centroid deviates from the pixel center. Only when centroid is well located in the pixel center and Gaussian radius $\sigma_r = 0.5$ pixel can the star image covers exactly 3×3 pixels (Wang et al., 2009).

Set the centroid coordinate of star image spot to be (x_m, y_n), floating-point, according to 2D Gaussian distribution, the gray value $g(i,j)$ of any pixel (i,j) is:

$$g(i,j) = \frac{A}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{(i-x_{\mathrm{m}})^2}{2\sigma_x^2} - \frac{(j-y_n)^2}{2\sigma_y^2}\right) \tag{1}$$

where A is the energy-gray coefficient, the amplitude of the Gaussian function, which is related to the total illumination, photoelectric sensitivity, integral time and the gain of hardware and software chain; σ_x and σ_y are two parameters of 2D Gaussian distribution, when the image spot is center-symmetrical about optical axis, there is $\sigma_r = \sigma_x = \sigma_y$.

If there is deviation between the centroid of the image spot and the center of the located pixel, the coordinates of the centroid are decimal, so the centroid coordinate (x_m, y_n) is of floating-point type. Assuming that the integer coordinates of the pixel where the centroid is located is (m, n), the deviations are: $\Delta x = x_m - m$, $\Delta y = y_n - n$. The Eq. (1) can be converted to:

$$g(i,j) = \frac{A}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{(i-m-\Delta x)^2}{2\sigma_x^2} - \frac{(j-n-\Delta y)^2}{2\sigma_y^2}\right) \tag{2}$$

The integer coordinate (m, n) is easy to determine, for example by determining the row and column number where the maximum of the gray accumulation appears,

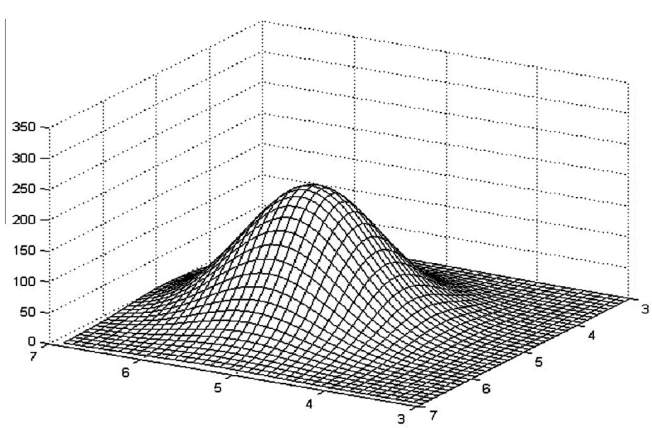

Fig. 1. A 2D Gaussian energy distribution contour of an image spot.

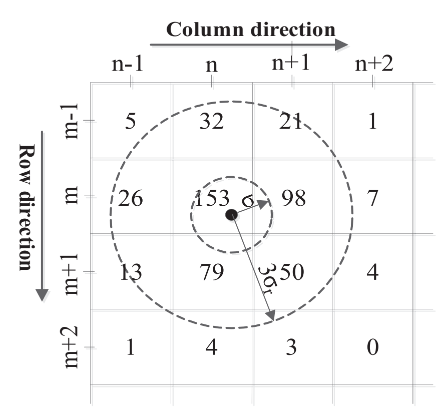

Fig. 2. Covering area of de-focus lens image spot.