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Short-arc tracklet association for geostationary objects

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Abstract

Measurement association and initial orbit determination is a fundamental task when building up a database of space objects. This paper proposes an efficient and robust method to determine the orbit using the available information of two tracklets, i.e. their line-of-sights and their derivatives. The approach works with a boundary-value formulation to represent hypothesized orbital states and uses an optimization scheme to find the best fitting orbits. The method is assessed and compared to an initial-value formulation using a measurement set taken by the Zimmerwald Small Aperture Robotic Telescope of the Astronomical Institute at the University of Bern. False associations of closely spaced objects on similar orbits cannot be completely eliminated due to the short duration of the measurement arcs. However, the presented approach uses the available information optimally and the overall association performance and robustness is very promising. The boundary-value optimization takes only around 2% of computational time when compared to optimization approaches using an initial-value formulation. The full potential of the method in terms of run-time is additionally illustrated by comparing it to other published association methods.

Keywords: Space situational awareness; Initial orbit determination; Tracklet association; Too-short-arcs; Admissible region

1. Introduction

In order to avoid accident-prone proximities of active satellites and uncontrolled space objects or eventually to remove space debris, space object catalogs must be maintained. The geostationary orbit is of special importance as it is intensively used for communication (e.g. Astra and Eutelsat satellites), navigation (e.g. the European Geostationary Navigation Overlay Service) and weather monitoring (e.g. Meteosat satellites). Additionally, it is special because of its far distance from the surface of the Earth. Due to the limited range capabilities of radar antennas, it is usually observed with optical telescopes. Considering the limited amount of telescopes trying to cover the complete orbital region, each object can only be tracked for a limited duration. The resulting short observation arcs, called tracklets, only contain incomplete state information and are therefore either associated to already cataloged objects (e.g. with the procedure described in Früh et al., 2009) or tested pairwise with other uncorrelated observations. The latter problem is approached in this work, i.e., two tracklets are examined whether they originate from the same object or not and, if they do, the common orbit solution is determined. This measurement association is a fundamental task during the catalog build-up phase for the initial location of space objects but also later for the relocation of lost ones.

1.1. Observations

Each measurement arc contains a series of right ascension (α) and declination (δ) values as measured by a

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topocentric observer. The information of the series, which is exploited for the association, is the line-of-sight $\mathbf{u}$ and its derivative $\dot{\mathbf{u}}$. It can also be represented by the angles and angular rates, commonly known as the attributable vector (c.f. Milani et al., 2004)

$$\mathbf{a} = (\alpha, \dot{\alpha}, \delta, \dot{\delta})^\top. \tag{1}$$

The use of the derivatives is only justified as long as measurement errors are small and the tracklet length is long enough for consistent catalog maintenance.

1.2. Problem formulation

The orbital motion of an object is described by six first order ordinary differential equations, where the vector form of the system is shown below

$$\dot{\mathbf{y}}(t) = \mathbf{f}(t, \mathbf{y}(t)) \quad \text{where} \quad \mathbf{y}(t) = \begin{pmatrix} \mathbf{r}(t) \\ \dot{\mathbf{r}}(t) \end{pmatrix}. \tag{2}$$

The state of the system is uniquely defined by six constraints, i.e. an orbit solution is either defined by an initial value

$$\mathbf{y}(t_1) = \begin{pmatrix} \mathbf{r}_1 \\ \dot{\mathbf{r}}_1 \end{pmatrix} \tag{3}$$

or by two boundary values

$$\mathbf{y}_{1:3}(t_1) = \mathbf{r}_1 \quad \text{and} \quad \mathbf{y}_{1:3}(t_2) = \mathbf{r}_2, \tag{4}$$

where in the first case (Eq. (3)) the position and velocity at one epoch must be provided and in the latter case (Eq. (4)) the positions at two observation epochs are needed. The subscript denotes the constrained elements of the state vector. The different components of the state vector can be described in terms of the observed variables. The geocentric position

$$\mathbf{r}(\rho) = \mathbf{r}_s + \rho\mathbf{u} \tag{5}$$

is dependent on the line-of-sight information, the sensor position $\mathbf{r}_s$, and the unobserved range ρ. The velocity

$$\dot{\mathbf{r}}(\rho, \dot{\rho}) = \dot{\mathbf{r}}_s + \rho\dot{\mathbf{u}} + \dot{\rho}\mathbf{u}, \tag{6}$$

additionally requires knowledge on the line-of-sight derivative, the sensor velocity, and the unobserved range-rate $\dot{\rho}$.

Therefore, one tracklet constrains the state of an object in four degrees of freedom with two unknown parameters $(\rho, \dot{\rho})$. Thus, to uniquely determine a state of an object, at least two observation arcs are required. Hence, an overdetermined system with eight known and four free parameters, namely the range and range-rate at both observation epochs is obtained. The tracklet association problem is illustrated in Fig. 1.

To date, various methods have been developed to approach the tracklet association problem. All methods hypothesize some of the unknown free parameters to obtain an orbit estimate. Then, these hypotheses are tested and either accepted or rejected. However, the methods differ in the way the orbit is represented, i.e. using the initial value or the boundary value formulation. Both formulations come with certain drawbacks and benefits, which will be discussed in Section 6.3. Table 1 gives an overview of current approaches using the different formulations and their relation to each other. They are recapitulated in the following.

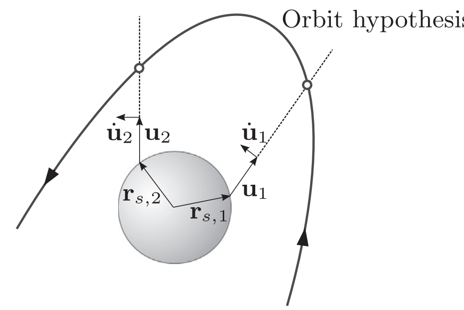

Fig. 1. Illustration of the two tracklet association problem: Shown are two line-of-sight vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ and their derivatives at different epochs and with different station coordinates $\mathbf{r}_{s,1}$ and $\mathbf{r}_{s,2}$. Furthermore, an orbit hypothesis connecting both observations is shown.

2. Initial-value method

When augmenting the tracklet information from the first observation epoch with the two free parameters $(\rho_1, \dot{\rho}_1)$, a full state is defined

$$\hat{\mathbf{y}}(t_1) = \begin{pmatrix} \mathbf{r}(\rho_1) \\ \dot{\mathbf{r}}(\rho_1, \dot{\rho}_1) \end{pmatrix}, \tag{7}$$

which can be used as an initial value to numerically or analytically integrate the equation of motion (2) to the second epoch. Consequently, the solution $\hat{\mathbf{y}}(t_2)$ can be computed. The hat over the state variable indicates that the orbital solution is a hypothesis and not necessarily the true one. The observation arc at the second epoch t_2 serves as a discriminator to decide whether a hypothesis is accepted or rejected.

Three different methods have been published which use this formulation. The solution space of the problem is limited to a so called admissible region in all these methods. Milani et al. (2004) and Tommei et al. (2007) suggest to restrict the range and range-rate space by allowing only solutions that orbit the Earth on stable orbits, i.e. the energy of the candidate orbits must be negative and the distance between the object and the observer must be above a certain limit. Maruskin et al. (2009) additionally defined bounds on apoapsis and periapsis radii. The latter assures that the candidate solution does not de-orbit within the next revolutions. If only a specific orbital region is of interest, the extent of the region can be furthermore reduced by specifying semi-major axis or eccentricity bounds (DeMars and Jah, 2013; DeMars et al., 2012).

2.1. Regular grid testing

DeMars et al. (2012) sample the admissible region of the first tracklet with multiple hypotheses on a regular grid to