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Satellite orbits design using frequency analysis

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Abstract

We present here a new method for the efficient computation of periodic orbits, which are of particular interest for low-altitude satellite orbits design in high degree/order, non-axisymmetric gravity models. Our method consists of an iterative filtering scheme, that is itself based on 'Prony's method' of frequency analysis, and is independent of the complexity of the gravity model. Applying this method to the case of a low-altitude lunar orbiter, we show that it converges rapidly, in all models and for all values of altitude and initial inclination studied. Thus, as demonstrated below, one could use it to correct the initial conditions of a desired mission orbit — usually defined within the framework of a simplified model (e.g. the 'J_2 problem') — ensuring minimal orbital eccentricity variations and, for very low altitudes, collision avoidance. At the same time, an accurate quasi-periodic decomposition of the orbit is computed, giving a measure of the periodic fluctuations of the orbital parameters.

Keywords: Satellite orbits; Orbit design; Frequency analysis

1. Introduction

When designing artificial satellite survey missions, one would like the chosen orbit to fulfill at least the following two criteria: (i) its geometry conforms to the necessities of the mission (e.g. low-altitude, near-circular, polar orbits for global surface mapping) and (ii) its size, shape and orientation do not change significantly on the time scale of the mission. The second criterion translates to minimal passive control (and fuel cost). Given the complex shape and gravitational field of most solar system objects, it is not *a priori* obvious which initial conditions could lead to an orbit that satisfies both requirements.

Satellite motion around a non-spherical primary has been extensively studied in the past, starting from the simple 'J_2 problem' (see Allan, 1970; Hughes, 1981; Jupp, 1988) where only the lowest-order zonal term is retained in the disturbing function. In the general 'zonal problem' — where the primary is considered axisymmetric and the disturbing function contains only zonal harmonics but of arbitrarily high order — a simple one degree-of-freedom (d.o.f.) problem can be constructed, if we average over the mean anomaly of the satellite[1] (see Kaula (1966) for a derivation of the Hamiltonian of this problem and De Saedeleer (2005) for an analysis). The fixed points of the resulting integrable dynamical system are the well-known *Frozen Orbits* (FOs); the inclination i, argument of pericenter (ω, or g in the usual notation of Delaunay variables) and the eccentricity e of these orbits are stationary, with $g = \pm 90^\circ$ and $e \sim \mathcal{O}(J_3/J_2)$ (see also Abad et al., 2009; Lara et al., 2009), while the longitude of the node (Ω, or h in Delaunay form) precesses at constant rate. Working

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[1] Assuming no resonances between the mean motion of the satellite and the rotational frequency of the primary can occur, as is the case e.g. for low-altitude lunar orbiters.

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along these lines, Knežević and Milani (1998) developed an analytical theory for the (averaged) motion of a low-altitude lunar orbiter, in high-order, axisymmetric gravity models. Near-polar FOs around the Moon were also studied by Carvalho et al. (2011), while (Delsate and Robutel, 2010) applied the same method to the case of Mercury.

For primaries with significant deviation from axial symmetry (such as the Moon), models of higher complexity are needed in order to adequately describe the secular dynamics of a satellite. The non-axisymmetric potential terms add one more degree of freedom in the problem, as they depend on the longitude of the node h and FOs do not generally exist. Instead, one should look for the next best thing, i.e. *Periodic Orbits* (POs) of this 2-d.o.f. system, for which i, g and e cannot be constant, but at least will suffer only small-amplitude, periodic variations (see Tzirti et al., 2010). These POs would appear as fixed points on a suitably chosen Poincaré section of the 4-d phase space. However, for very complex gravity models (e.g. non-axisymmetric model of degree higher than $n > 10$), applying the standard method of differential correction to the equations of motion in order to find POs becomes extremely cumbersome.

In a recent paper (Tzirti et al., 2014), we studied the secular motion of a satellite around the Moon, using a novel frequency analysis (FA) algorithm that we call *Prony's method*. This allowed us to obtain a global view of the secular dynamics, for several different models of the lunar potential. The method is quite efficient computationally, as one needs as input only a short segment of the trajectory, and can be easily applied to large sets of initial conditions. The results can be viewed in the form of 2-d frequency/amplitude maps of the initial conditions space (e.g. the $i - e$ plane, for a given altitude and orientation in g, h). By inspecting these maps, one can find approximately the location of a PO of interest (e.g. a near-polar PO). Repeating the same analysis on a finer grid of initial conditions set around this 'first-guess' of the PO, one can actually approximate the PO with an accuracy that is enough e.g. so that the resulting orbit does not drift away on a time-scale ~30–50 months. It goes without saying of course that this 'brute-force' technique can be computationally quite expensive, especially if extremely accurate models of the gravitational potential are considered (i.e. designing a real mission). Still, using the FA map as a guide is certainly better than blind search in a 4-d elements space. Note that, for gravity models of degree $n = 7$ or higher, POs can be substantially displaced in (e, g) with respect to the FOs of the $n = 3$ or higher axisymmetric problem (Tzirti et al., 2014) that could be easily computed analytically.

In this paper we extend the use of our FA method, by developing a computationally efficient iterative scheme that allows us to locate the initial conditions of a PO with very high accuracy, starting from a first guess that needs not even be close to the actual PO. Applying this method to the Moon, we show that it converges fast for all initial values of orbital inclination and altitude tested. In the following sections we first describe the essentials of Prony's FA method and the lunar gravity models used here. We then describe our 'filtering scheme' and its convergence properties. Maps of POs for different altitudes are given, for the case of the Moon. Finally, conclusions and discussion are presented in the last section of the paper.

2. Frequency analysis and Prony's method

To characterize satellite orbits and their "distance" to Periodic Orbits, it is necessary to be able to separate them into different periodic components. The scope of frequency analysis (FA) is precisely to obtain the decomposition of a quasi-periodic signal $u(t)$ into a set of p different periodic waves

$$u(t) = \sum_{k=1}^{p} \alpha_k \, \mathrm{e}^{\,\mathrm{i}\,2\pi\nu_k t}, \tag{1}$$

by determining all frequencies ν_k (and their (complex) amplitudes α_k), using a sample of the signal of finite duration T, i.e. a finite number N of discrete values of $u[n]$, separated by a constant time interval Δt

$$u[n] \equiv u(n\,\Delta t) \quad n = 0, \ldots, N-1 \tag{2}$$

$$= \sum_{k=1}^{p} \alpha_k \, \rho_k^n \quad \rho_k \equiv \mathrm{e}^{\,\mathrm{i}\,2\pi\nu_k \Delta t}. \tag{3}$$

There exist different ways of computing the spectrum of $u[n]$, as was described in detail in Tzirti et al. (2014). The most common method is that of computing numerically the Fourier transform of $u[n]$; this is the basis of the more refined and well-renowned `NAFF` algorithm of Laskar (1990) (see also Laskar, 2005 or Laskar et al., 1992). The Fourier/`NAFF` method gives very good results, if the frequencies present in the signal are well separated. A known shortcoming of this method is that, to compute all frequencies accurately, it requires the sample duration T to be at least equal to the longest period present in the signal (like all Fourier based methods).

In Tzirti et al. (2014) we applied a different method, which we call 'Prony's method'; we repeat here the essentials and refer the reader to that paper for more details. The method is in fact the complex version of Prony's original method for real exponentials (de Prony, 1795) (see a modern description in Hamming, 1973; Hildebrand, 1987; Kay, 1988 or Noullez, 2009), which was originally used by Prony to describe the time evolution of the pressure during the expansion of gases. The essence of the method is that any discretely sampled signal that is a simple sum of p exponentials as in (3) obeys a *linear* constant coefficients difference equation of order p

$$u[n] + a_1\, u[n-1] + \cdots + a_p\, u[n-p] = 0, \tag{4}$$

and the p complex "resonances" ρ_k are the roots of the characteristic polynomial