Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 57 (2016) 1189–1196

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

www.elsevier.com/locate/asr

Frequency analysis of the non-principal-axis rotation of uniaxial space debris in circular orbit subjected to gravity-gradient torque ☆

Hou-Yuan Lin [a,b,c], Chang-Yin Zhao [a,b,*], Ming-Jiang Zhang [a,b,c,1]

[a] Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
[b] Key Laboratory of Space Object and Debris Observation, Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
[c] University of Chinese Academy of Sciences, Beijing 100049, China

Received 9 April 2015; received in revised form 10 December 2015; accepted 22 December 2015
Available online 29 December 2015

Abstract

The non-principal-axis rotational motion of uniaxial space debris can be decomposed into periodic motions associated with two frequencies: the polhode frequency of the space debris rotating around the symmetry axis, and the tumbling frequency of the symmetry axis rotating around the angular momentum. To determine from optical measurements the rotational motion of upper rocket stages in circular orbits subjected to gravity-gradient torque, the evolutions of these two frequencies need to be analyzed. Taking into account only the long-term changes in the long-period variables, the differential equations of the non-principal axis rotational motion of the uniaxial space debris are averaged and reduced, from which the evolutions of the polhode and tumbling frequencies are then obtained analytically. The theoretical results are verified by numerical simulations of the diffuse reflection model. The frequencies in the variation of the reflected light intensity in the simulation are analyzed using the frequency map analysis (FMA) method. Errors of these results are found to be less than 1%. Based on the theoretical expressions, the rotational state of the uniaxial space debris can be estimated in the simulation without any prior information except the orbital parameters. A series of state variables are estimated, including the ratio of the moments of inertia about the transverse axis and the symmetry axis, the instantaneous rotation velocity, the orientation of the angular momentum, and the precession cone of the symmetry axis.

Keywords: Active debris removal; Rotational motion of space debris; Frequency analysis; State estimation

1. Introduction

Active debris removal is an effective method to remedy the growing number of space debris objects (Liou, 2011). One of the key tasks in such removal is stabilizing non-cooperative rotational targets (upper rocket stages mainly) and then capturing them. Thus the rotational state of the target must be estimated first (Liou, 2011; Ojakangas et al., 2012). The rotational characteristics of space debris can be determined through the optical variability in its diffuse reflection (Giese, 1963). The often-adopted symmetric cylinder model fixes the rotational axis of the target on one of the principal axes of inertia (Giese, 1963; Williams, 1979a; Santoni et al., 2013). Williams (1979a,b) determined the orientation of the symmetry axis through the ratio of the maximum to minimum values of the optical variability. Santoni et al. (2013) acquired better results with the same method but higher precision data. Yanagisawa and

☆ This template can be used for all publications in Advances in Space Research.

* Corresponding author at: Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China. Tel.: +86 25 83332251; fax: +86 25 83332252.

E-mail addresses: linhouyuan@pmo.ac.cn (H.-Y. Lin), cyzhao@pmo.ac.cn (C.-Y. Zhao), mjzhang@pmo.ac.cn (M.-J. Zhang).

[1] Tel.: +86 25 83332248; fax: +86 25 83332252.

http://dx.doi.org/10.1016/j.asr.2015.12.036
0273-1177/

Kurosaki (2012) built a tri-axial ellipsoid model that rotates about the shortest principal axis to estimate the rotational state through the observation data. However, the assumption that the rotational axis of the target is fixed on the principal axis in the state estimation through optical observation often causes large errors in the results (Williams, 1979b; Yanagisawa and Kurosaki, 2012). When the target is rotating about a non-principal axis, it is complicated to estimate the rotational state from brightness variation only, because there are two frequencies: the polhode and the tumbling frequencies (Hitzl and Breakwell, 1971; Markeyev, 2006). Furthermore, taking into account gravity-gradient torques, these two frequencies depend on the perturbation (Hitzl and Breakwell, 1971), which makes it even harder in the estimation of the rotational state.

The determination of the evolutions of these two frequencies perturbed by gravity-gradient torques is essential in estimating the rotational state of the target. In the past, studies on the effect of gravity-gradient torque on the rotational motion of the target mainly focused on the secular evolution of the angular momentum through canonical transformations. Crenshaw and Fitzpatrick (1968) applied Poisson's equation to acquire analytical expressions of the general rotational motion of the uniaxial satellite in space. Subsequently, Liu and Fitzpatrick (1975) extended the equation to a tri-axial satellite. Holland and Sperling (1969) averaged the gravity-gradient torque equation to obtain the long-term changes in the rotational angular momentum vector. Hitzl and Breakwell (1971) discussed the non-resonance and near-resonance effects of the gravity-gradient perturbations. Cochran (1972) studied the effect of the gravity-gradient torque using Lie series, in which the orbital eccentricity and the orbital precession caused by the Earth's oblateness are considered. Nevertheless all results developed from the above may be applied with difficulty for observational applications.

In this paper, we analyze the medium-term evolutions of the polhode and tumbling frequencies of uniaxial space debris subjected to gravity-gradient torques for the estimation of the rotational state. In our analysis, we make some assumptions and impose constraints. First, the target (upper rocket stages) is assumed to be a symmetric rigid body, where two of the principal moments of inertia are identical. Second, according to studies on eddy current torque, the rotation period of the conducting shell-shape debris may increase to hundreds of seconds in a year (Wilson, 1977; Praly et al., 2012; Lin and Zhao, 2015). Hence, we choose the rotation period of around one hundred seconds for our target and in this case the effect of the gravity-gradient torque is more significant. Moreover, changes in spin speed and axial direction caused by the dissipation are neglected in the whole process (Williams and Meadows, 1978). Section 2 presents the equations describing the non-principal axis rotational motion of the uniaxial space debris subjected to gravity-gradient torques, and then they are averaged and reduced to extract the analytical evolutions of these two frequencies mentioned above. In Section 3, theoretical results are verified using the diffuse reflection model in numerical simulations. Based on the theoretical expressions, the rotational state of the uniaxial space debris can be estimated from the simulations.

2. Frequency analysis of spin motion

2.1. Rotational equations of uniaxial space debris in orbit

As described in Crenshaw and Fitzpatrick (1968), we introduce four rectangular coordinate systems with their origins fixed at the centre of mass of the uniaxial space debris (see Fig. 1). The orientation of the $OX^*Y^*Z^*$ system is fixed in the space, for which the fundamental plane is the equatorial plane of the Earth, and the OX^* axis points to the vernal equinox. The $OX^0Y^0Z^0$ system is the orbital coordinate system of the uniaxial space debris, and the OX^0 axis is in the direction of the ascending node. The $OX'Y'Z'$ system is the body-fixed system of the uniaxial space debris with the symmetry axis OZ'; the $OXYZ$ system is a system associated with the angular momentum vector $\boldsymbol{h} = h\hat{\boldsymbol{h}}$ of the uniaxial space debris, whose OZ and OX axes are directed respectively along the vector $\boldsymbol{h}$ and the line of nodes formed by the XY and $X'Y'$ planes. The $OX'Y'Z'$ system can be obtained from the $OXYZ$ system by two successive rotations through angles θ' and φ'; and the $OXYZ$ system can also be obtained from the $OX^0Y^0Z^0$ system by three rotations through successive angles ψ_H, θ_H, and φ_H.

Based on the theory of Crenshaw and Fitzpatrick (1968), the equations of the rotational motion of the

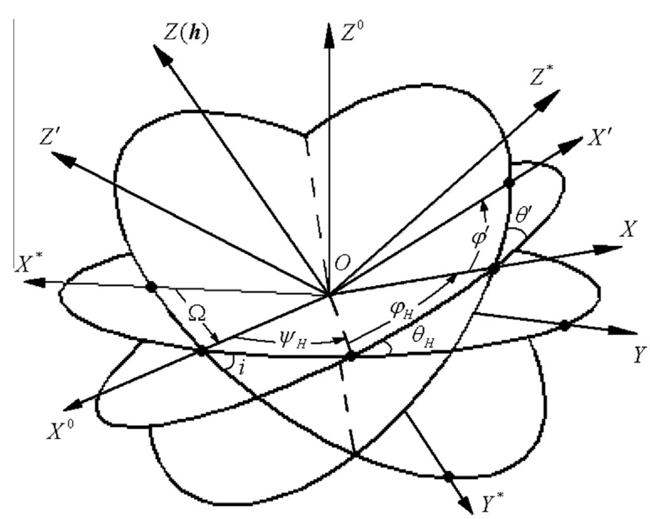

Fig. 1. Coordinate systems, as described in Crenshaw and Fitzpatrick (1968). The orientation of the $OX^*Y^*Z^*$ system is fixed in the space, for which the fundamental plane is the equatorial plane of the Earth, and the OX^* axis points to the vernal equinox. The $OX^0Y^0Z^0$ system is the orbital coordinate system of the uniaxial space debris, and the OX^0 axis is in the direction of the ascending node. The $OX'Y'Z'$ system is the body-fixed system of the uniaxial space debris with the symmetry axis OZ'; the $OXYZ$ system is a system associated with the angular momentum vector $\boldsymbol{h} = h\hat{\boldsymbol{h}}$ of the uniaxial space debris, whose OZ and OX axes are directed respectively along the vector $\boldsymbol{h}$ and the line of nodes formed by the XY and $X'Y'$ planes.