Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 56 (2015) 907–915

ADVANCES IN SPACE RESEARCH (a COSPAR publication)

www.elsevier.com/locate/asr

Regional TEC dynamic modeling based on Slepian functions

Mohammad Ali Sharifi, Saeed Farzaneh *

Department of Surveying and Geomatics Engineering, University College of Engineering, University of Tehran, North Kargar Ave., P.O. Box 11365-4563, Tehran, Iran

Received 10 January 2015; received in revised form 14 May 2015; accepted 15 May 2015
Available online 22 May 2015

Abstract

In this work, the three-dimensional state of the ionosphere has been estimated by integrating the spherical Slepian harmonic function and Kalman filter. The spherical Slepian harmonic functions have been used to establish the observation equations because of their properties in local modeling. Spherical harmonics are poor choices to represent or analyze geophysical processes without perfect global coverage but the Slepian functions afford spatial and spectral selectivity. The Kalman filter has been utilized to perform the parameter estimation due to its suitable properties in processing the GPS measurements in the real-time mode. The proposed model has been applied to the real data obtained from the ground-based GPS observations across some portion of the IGS network in Europe. Results have been compared with the estimated TECs by the CODE, ESA, IGS centers and IRI-2012 model. The results indicated that the proposed model which takes advantage of the Slepian basis and Kalman filter is efficient and allows for the generation of the near-real-time regional TEC map.

Keywords: Ionospheric delay; Slepian functions; Kalman filter; Total electron content

1. Introduction

Since the selective availability in GPS was removed in 2000, the ionospheric delay has become a pronounced error in GPS positioning and navigation. In order to correct this error in the single-frequency GPS positioning, one way is to model the ionosphere using GPS observations and then the GPS users make use of this model to correct the ionospheric delay. This model should be able to predict the TEC corrections with high accuracy so that the GPS users can mitigate the ionospheric effects in the real time.

Over the last two decades, several groups have developed different mathematical models to monitor the state of the ionosphere using data from the GPS satellites. GPS-based ionospheric models can be spatially classified into two categories: the two-dimensional (2-D) or three-dimensional (3-D) models of the electron density. The 3-D modeling is based on latitude, longitude and height, in which the slant total electron content (STEC, i.e. the number of electrons presented in a column of the 1 m^2 cross-section and extended along the ray-path of the signal between the GPS satellites and the ground receiver) measurements are inverted into electron density distribution using tomographic approaches (for a comprehensive overview on various techniques of ionospheric density modeling, see e.g., Garcia-Fernandez, 2004). As the permanent GPS reference stations are not globally distributed in a uniform manner and do not supply high vertical resolution for ionospheric tomography (Garcia-Fernandez et al., 2003), the additional observations derived from ionosondes, satellite altimetry or GPS receivers on Low-Earth-Orbiting (LEO) satellites have been considered in several 3-D studies (Stolle et al., 2003; Schmidt et al., 2007; Zeilhofer, 2008). Due to the fact that the spatial

* Corresponding author.
E-mail addresses: sharifi@ut.ac.ir (M.A. Sharifi), farzaneh@ut.ac.ir (S. Farzaneh).

http://dx.doi.org/10.1016/j.asr.2015.05.024
0273-1177/

and temporal distributions of the additional observations are usually limited, they can only be applied where or when the observations are available. Therefore, like this study the majority of the GPS-based studies have utilized the 2-D modeling, which are well addressed in Wielgosz et al. (2003), Mautz et al. (2005) and Liu et al. (2008). Since the relatively long-term TEC predictability is highly crucial for navigation and space applications; and due to the fact that the global ionospheric models support the global TEC predictions and are not suitable for local applications; and the above-mentioned regional models could not predict the long-term of TEC, the models using the past observations to predict the future TEC values are needed (Liu et al., 2011).

Recently, a new method called saptio-spectral localization for spherical harmonic analysis has been developed for the problems in which the data is measured over some portion of the sphere, such as the north hemisphere, a spherical cap, continents and so on. The spherical harmonic functions can effectively be used to represent the target function on condition that the modeled area has covered the whole sphere, and that the data is distributed regularly. This basis is prone to errors since it is not orthogonal over the partial area at all. Therefore, the construction of a local spherical harmonic basis that is orthogonal over the studied area of the sphere is required for the study of the local ionospheric modeling. The so-called spatio-spectral concentration problem can be solved by determining an orthogonal family of the strictly band-limited functions that are optimally concentrated within a closed region of the sphere or by considering an appropriate orthogonal family of the strictly space-limited functions optimally concentrated in the spectral domain (Simons et al., 2006). Our approach uses the spatio-spectral localization with Slepian basis functions (Simons et al., 2006). The Slepian functions are a set of band-limited functions that have their energy optimally concentrated inside a spherical cap (Simons et al., 2006). It has been proved that it is an effective method to calculate the localized power spectrum using Slepian tapers (Wieczorek and Simons, 2005).

In this study, the spatial modeling of VTEC was done by the Slepian function and in order to achieve the near-real-time estimations, the Kalman filter technique was applied for temporal variations.

2. TEC determination

In this study, a method of measuring TEC directly from the differential code delay and carrier phase measurement on both the L1 and L2 frequencies was used. For this purpose, the carrier-to-code leveling process method (Ciraolo et al., 2007; Nohutcu et al., 2010) was used. The necessary formula can be extracted from Sharifi and Farzaneh (2013). Once the VTEC values were computed for each satellite, they were combined in the least-squares sense as discussed in detail by Arikan et al. (2007). These values were then used for the near-real time modeling.

3. Spherical Slepian functions

In this research the observation matrix whose elements are constant throughout the process was determined using the spatio-spectral localization with Slepian basis functions (Simons et al., 2006). The spherical Slepian basis, a set of the band-limited functions which have the majority of their energy concentrated by optimization inside an arbitrarily defined region, provides an efficient way for the analysis and representation of the spatio-spectrally localized geophysical signals (Simons et al., 2006; Harig and Simons, 2009). The Slepian basis functions can be defined as:

$$g_\alpha(\hat{\boldsymbol{r}}) = \sum_{l=0}^{L}\sum_{l=-m}^{m} g_{\alpha,lm} Y_{lm}(\hat{\boldsymbol{r}}) \tag{1}$$

where $Y_{lm}(\hat{\boldsymbol{r}})$ is the spherical harmonic basis function of degree l and order m and $g_{\alpha,lm}$ is the Slepian function coefficients which are defined as:

$$g_{\alpha,lm} = \int_\Omega g_\alpha(\hat{\boldsymbol{r}}) Y_{lm}(\hat{\boldsymbol{r}}) d\Omega \tag{2}$$

These functions maximize their energy within the region of interest R according to:

$$\lambda = \max_G \frac{\int_R {g_\alpha}^2(\hat{\boldsymbol{r}}) d\Omega}{\int_\Omega {g_\alpha}^2(\hat{\boldsymbol{r}}) d\Omega} \tag{3}$$

The $0 \leqslant \lambda \leqslant 1$ is a measure of the spatial concentration. As shown by Simons et al. (2006), Eq. (3) is reduced to the matrix eigenvalue equation. This means that the Slepian coefficients are found by solving the following eigenvalue equation:

$$\sum_{l'=0}^{L}\sum_{m'=-l'}^{l'} D_{lm,l'm'} g_{l'm'} = \lambda g_{lm} \tag{4}$$

where the elements of $D_{lm,l'm'}$ are the products of spherical harmonics integrated over the region R:

$$\int_R Y_{lm} Y_{l'm'} d\Omega = D_{lm,l'm'} \tag{5}$$

It is noteworthy that in this research, the Slepian functions have been preferred over the Principal Component Analysis (PCA) or its equivalent Singular Value Decomposition (SVD) methods to estimate the integral in Eq. (5). In fact, PCA/SVD represents a fairly well performance to find linear sub-spaces that adequately reconstruct the dominant variability of data sets rather than basis-function energy (Forootan, 2014). However, several studies show that estimation of eigenvectors, that represent the main variance contained in the samples, is domain-dependent (Richman, 1986). This might represent a limitation in estimating 4D maps when changing the spatial domain or considering another time-frame.