Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 56 (2015) 929–940

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

www.elsevier.com/locate/asr

Stochastic transfer of polarized radiation in finite cluttered atmospheric media with dual statistical analysis

M. Sallah

Theoretical Physics Research Group, Physics Department, Faculty of Science, Mansoura University, Mansoura P.O. Box 35516, Egypt

Received 22 January 2015; received in revised form 9 May 2015; accepted 13 May 2015
Available online 21 May 2015

Abstract

The stochastic transfer of polarized radiation in a finite cluttered atmospheric medium (e.g. clouds) is investigated, the solution being presented for arbitrary absorption and scattering cross sections. The extinction function of the medium is assumed to be a continuous random function of position, with fluctuations about the mean taken as Gaussian distributed. The deterministic analytical solution is obtained by using Pomraning–Eddington technique with weight function method. Two correlated random variables appear in the solution, namely the optical space variable and the medium optical thickness. The dual Gaussian probability density function of these two random variables is derived, from which the ensemble-averaged solution is calculated for an arbitrary correlation function. The first and the second statistical moments of some quantities of interest, such as radiant energy, net flux, reflectivity and transmissivity, are obtained. Numerical calculations are performed for specular-reflecting boundaries and an incident flux of radiation upon the medium from one side ($x = 0$) and with no flux from other side ($x = L$).

Keywords: Stochastic polarized-radiative-transfer; Dual Gaussian statistics; Cluttered atmospheric finite media; Pomraning–Eddington technique

1. Introduction

Long time ago, Rayleigh formulated his scattering of solar radiation which can be considered with or without polarization. Therefore, Chandrasekhar (1960) formulated the transfer equation of the solar radiation. Including the scattering with polarization, the scalar radiative transfer equation should be replaced by its vector form in terms of the four Stokes parameters. Many scientists tried to solve this polarized-radiative transfer equation either analytically or numerically (Sallah, 2007a). For instance, Landi Degl'Innocenti and Landi Degl'Innocenti (1985) presented a formalism to obtain an analytical solution for the radiative transfer equation for the polarized radiation when the Mueller matrix for the Stokes parameters is constant along the ray-path. Such formalism is applied to find an analytical approximation of the Stokes parameters for a certain chromospheric line. Evans and Stephens (1991) dealt with a numerical model that solved the polarized radiative transfer equation for a plane-parallel, vertically-inhomogeneous scattering atmosphere. They had taken into account the full polarization characteristics of randomly-oriented particles with any shape having a plane of symmetry. Later, Evans and Stephens (2010) discussed the significance their earlier work (Evans and Stephens, 1991) to show how their two plane-parallel polarized radiative transfer codes came about, how their codes had been used, and the related recent developments in polarized-radiative transfer modeling. Landi Degl'Innocenti (1996) continued to obtain an analytical description of the polarized radiative transfer by using the approach of the density matrix.

There has been an increasing interest in the use of polarization in imaging through cluttered media (stochastic

E-mail address: msallahd@mans.edu.eg

http://dx.doi.org/10.1016/j.asr.2015.05.019
0273-1177/

media). Optical imaging through clouds and fog, imaging in biological media, and microwave detection in clutter can benefit from the additional information provided by the polarization characteristics (Ishimaru et al., 2001). Polarization pulse propagation in cluttered media has also received attention, most of the studies being based on Monte Carlo calculations (Bruscaglioni et al., 1993; Lewis et al., 1999; Kim et al., 2001). Also, the observations of polarized radiation give considerable valuable information about stars with large gaseous and dust envelopes (Raveendran, 2002; Polyakova, 2003), reflection nebulae of different kinds (King et al., 2002; Rodrigues et al., 2003), interstellar medium (Leonard et al., 2002), active galaxies (Hagen-Thorn et al., 2002), etc. (see Freimanis (2005)). Freimanis (2014) has studied the polarized radiative transfer equation in homogeneous space by considering its differential operator obeying Euclidean geometry for several orthogonal curvilinear coordinate systems of astrophysical interest. The medium is assumed to be statistically homogeneously filled with polydisperse particles.

Due to the importance of studying atmospheric polarization and its radiative-transfer parameters, computer modeling of light scattering by aggregates and their light scattering characteristics, specifically polarization, change with phase angle and wavelength are presented by Kolokolova and Mackowski (2012). They developed of a new version of multi sphere T-matrix code for parallel computing. Their modeling results were applied to the comet polarimetric observations to check if large aggregates dominate information of light scattering by comet dust.

In this work, the problem of the transfer of polarized-radiation but in a cluttered stochastic atmospheric medium (e.g. clouds) is studied. So, it is reasonable to characterize the medium in a statistical sense. This requires the specification of the probability distribution of the fluctuating property. In this sense, two statistical models are particularly useful for obtaining the average solution for a large class of problems. In one of these, the medium is assumed to consist of two randomly mixed immiscible components, with the mixing obeying Markovian statistics. The review by Pomraning (1991) summarizes the work on this subject and provides an interesting list of references to some earlier work. In this case, the random property was assumed to have only two values corresponding to the value of each component. For instance, Sallah (2007b) studied the transfer of polarized radiation in a two-component stochastic medium described by binary discrete Markovian statistics. In the second model (which is the considered model in this work), the fluctuating property is assumed to be continuos and have a normal distribution. This requires to specify the probability distribution of the fluctuating property with its spatial and temporal correlations, etc. In this statistical model, the fluctuating property is assumed to have a Gaussian distribution, characterized by a mean value, variance, and correlation function. Here, we can quote some papers by Prinja (1993, 1996), Prinja and Pomraning (1995), and Prinja and Gonzalez-Aller (1996). In our former papers (Selim and Sallah, 2008a,b), we had dealt with Gaussian statistics in the 2-dimensional geometry for a semi-infinite and finite media. The physical property is allowed to vary continuously along each direction in this model. Both models enjoyed considerable popularity in diverse applications in several fields of physics, including nuclear technology, astrophysics, and climatology.

Therefore, our previous achievements (Selim and Sallah, 2008a,b; Sallah, 2014) are extended here to obtain a statistical investigation for the average solution of the stochastic transfer of polarized radiation in a cluttered atmospheric finite plane-parallel medium using the dual-Gaussian statistics. The advantage of this model is that it is easy to characterize, requiring only the mean, variance, and two-point correlation function to specify the distribution of random field completely.

The layout of this paper is as follows: The mathematical model of the polarized-radiative transfer has been formulated in Section 2. Then, the deterministic solution is obtained analytically by using the Pomraning–Eddington technique to obtain a closed form for the polarized-radiation intensities for a finite participating medium. The radiant radiation energy and the net radiation flux, as well as the reflectivity and the transmissivity at the boundaries, are obtained analytically. The medium is treated with a given value of the angular-dependent incident flux of radiation impacting on one of the specular-reflecting boundaries. Section 3 presents the statistical analysis of the problem, where the stochasticity of the extinction function, $\sigma(z)$, its mean $\bar{\sigma}$, variance η_σ, and autocorrelation function are clarified. When going from the geometrical space to the optical space, we focus attention on two random variables, x (optical variable) and L (medium optical thickness). The mean and the correlation matrix Q of these two random variables are calculated using a given exponential autocorrelation function, $W_\sigma(z)$. Then, the ensemble average solution is obtained to give the ensemble-averaged quantities of interest, as well as the second moments of them. Section 4 is quoted for the numerical results of the average of radiant radiation energy, net radiation flux, reflectivity and transmissivity for different degrees of polarization with varying the statistical parameter β that characterizes the stochastic cluttered medium.

2. Math. model and deterministic solution

The starting point of this analysis is the monoenergetic radiative transfer equation in a planar finite medium, which is given by Chandrasekhar (1960) and Pomraning (1991)

$$\left[\mu\frac{\partial}{\partial z}+\sigma(z)\right]\Psi(z,\mu)=\frac{\sigma_s(z)}{2}\int_{-1}^{1}P(\mu,\mu')\Psi(z,\mu')d\mu',\qquad(1)$$

$$0\leqslant z\leqslant Z_0,\quad -1\leqslant\mu\leqslant+1$$