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An econometric investigation of the sunspot number record since the year 1700 and its prediction into the 22nd century

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Abstract

Solar activity, as measured by the yearly revisited time series of sunspot numbers (SSN) for the period 1700–2014 (Clette et al., 2014), undergoes in this paper a triple statistical and econometric checkup. The conclusions are that the SSN sequence: (1) is best modeled as a signal that features nonlinearity in mean and variance, long memory, mean reversion, 'threshold' symmetry, and stationarity; (2) is best described as a discrete damped harmonic oscillator which linearly approximates the flux-transport dynamo model; (3) its prediction well into the 22nd century testifies of a substantial fall of the SSN centered around the year 2030. In addition, the first and last Gleissberg cycles show almost the same peak number and height during the period considered, yet the former slightly prevails when measured by means of the estimated smoother. All of these conclusions are achieved by making use of modern tools developed in the field of Financial Econometrics and of two new proposed procedures for signal smoothing and prediction.

Keywords: Solar cycles; Financial Econometrics; Time-series decomposition; Harmonic analysis

1. Introduction

The study of solar activity, in terms of emission of heat, electromagnetic waves and particles, and commonly measured as sunspot numbers (SSN), is very important in several disciplines such as Astrophysics, Earth Sciences and Climatology.

The pattern of the SSN time series (alternatively herein referred to as "signal") has been thoroughly analyzed and at times also reconstructed by many distinguished scientists since the times of Galileo, among which, for ease of space, only a few may be named. SSN has been analyzed from both the statistical and the econometric viewpoint (Yule, 1927; Granger, 1957; Yoon, 2006), and has undergone thorough data collection and reconstruction of its long-term past, as far as back as the year 1610 (Schove, 1948; Eddy, 1976; Hoyt and Schatten, 1993; Usoskin, 2013), as well as cyclical behavior analysis (Schwabe, 1843; Hathaway et al., 1994, 2002; Svalgaard et al., 2005; Usoskin et al., 2000, 2007; Svalgaard and Hudson, 2010; Hathaway, 2010).

Moreover, the likely climatological effects of the SSN on planet Earth have been inspected (Petrovay, 2010; Scafetta, 2014), alongside with cyclical short- and long-run prediction by means of calibration-based computational techniques (Thompson, 1993; Hoyt and Schatten, 1998; Mörner, 2010, 2011; Love and Rigler, 2012; Solheim et al., 2012, Solheim, 2013; NASA, 2014; Krainev, 2013), by means of statistical extrapolation (Lomb and Andersen, 1980; Hiremath, 2007, 2008; Brajša et al., 2009; Barnhart and Eichinger, 2011; Werner, 2012; Lomb, 2013), and by means of harmonic-model analysis (Scafetta, 2012, 2013). Similar predictive research has been applied to solar proxies, among which Total Solar Irradiance (Velasco Herrera et al., 2011) and the Radio Flux (NOAA, 2015).

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All of the cited articles have produced interesting and illuminating insights into the working mechanism of the SSN and also into their expected trends, especially those regarding the second part of the current cycle 24 and the very next cycles (e.g. Solheim et al., 2012; NASA, 2014).

Given such multi-faceted evidence on the pattern of SSN and on its causes, the present paper attempts to shed some additional light onto the results so far achieved in the literature by means of a three-pronged statistical-based strategy that properly: (1) classifies the annual SSN series according to up-to-date Financial Econometrics tools; (2) tests for the solar flux-transport dynamo hypothesis; (3) exhibits the major time-series descriptive features and produces a smoothed nonlinear trend (henceforth denominated the "smoother") as well as a long-run prediction of the SSN. The two are obtained by means of appropriate techniques which are respectively named Automated Smoothing Method (ASM) and Optimal Cycle Bootstrapping (OCB).

Section 2 treats the theoretical and empirical aspects of time-series decomposition and of both nonlinearity and stationarity, whereas Section 3 sequentially treats two issues: solar dynamics in the context of harmonics and the new proposed ASM technique. Section 4 is devoted to the theoretical and applied aspects of OBC and subsequently exhibits the empirical results of the one century-ahead SSN prediction. Section 5 concludes.

2. Time-series decomposition, nonlinearity and stationarity tests

In this Section the problematic issue of nonlinearity is analyzed in the context of random signals whose classical decomposition features are illustrated. The task is performed both theoretically and empirically. In the latter case, some descriptive statistics and some recent nonlinearity and stationarity tests abridged from Financial Econometrics are supplied.

2.1. General time-series decomposition

Nonlinearity is rather common in time series of natural phenomena, and generally features some or all of the following characteristics: non-normality, asymmetric cycle distribution, bimodality, nonlinear relationship among lagged variables and between these and the error term, time irreversibility, and variation of predictive performance (Yule, 1927; Engle 1982; Bollerslev, 1986; Tiao and Tsay, 1994; Huang et al., 1998, 2003; Huang, 2007; Lineesh and Jessy, 2010). Moreover, parameter estimation by means of Ordinary Least Squares (OLS) is likely to be biased (Winschel and Krätzig, 2010), and the Augmented Dickey-Fuller (ADF) test for stationarity (Said and Dickey, 1984; Elliott et al., 1996) no longer applies. The ADF test must be replaced by a procedure specifically designed for nonlinear phenomena (Harvey et al., 2008; Kapeitanos and Shin, 2002; Kapeitanos et al., 2003).

In any case, the classical time-series additive decomposition for nonlinear series ordinarily applies (Cramér, 1961). Specifically, for the discrete time notation $t = \in [1, T]$, a signal is defined as a chronological sequence of real-valued random numbers expressed as $\{X_t\}_{t=1}^{T} \sim \mathrm{IID}(\bar{X}, \sigma_X^2)$. The signal may be modeled as an autoregressive moving average (ARMA) or autoregressive conditional heteroskedasticity (ARCH) process (Box and Jenkins, 1970; Engle, 1982). More generally, the signal may be modeled as the sum of signals of unknown distribution such that the observed signal X_t is

$$X_t = X_t^* + S_t + \varepsilon_t; \quad X_t^* = \mathrm{E}(X_t|\Omega_{t-1}) \tag{1}$$

where $\Omega_{t-1} = \{X_{t-j}\}_j^J$ is the information set available at $t-j$, $j \in [1, J]$, $J \leqslant T$ is a maximum preselect lag, and $\varepsilon_t \sim \mathrm{IID}(0, \sigma_\varepsilon^2)$. The first term of Eq. (1) is the slow-mode component of the signal, either a linear trend or a smoother, or also a combination of the two. The second component S_t is the periodical or quasi-periodical seasonal cycle, and the last is a random Gaussian fast mode. Obviously, independent of S_t, if the linear and/or the smoothed trends are not time-dependent, the first component zeroes out in mean and the process described by Eq. (1) collapses to ε_t.

All components of X_t are unobservable and must be retrieved from the raw dataset by means of appropriate computational methods. In particular the slow-mode term, if not solely represented by a linear trend, requires applying the smoothing procedures described in Section 3. In addition, if S_t is a high- or low-frequency phenomenon, it may be entrenched into either ε_t or X_t^* and thus difficult to disentangle from either component.

The signal may be defined strongly nonlinear if both modes are nonlinear, and weakly nonlinear if either mode is nonlinear (Fan and Yao, 2003). In such context, the traditional ARMA model with lags p and q, commonly denoted as the ARMA(p,q) process, frequently utilized to produce the slow mode, must be modified in the presence of a nonlinear relationship between the observable and the shock. Hence we specify the information set attached to Eq. (1) as $\Omega_{t-1} = \{\hat{X}_{t-j}\}_j^J$, where $\hat{X}_t = g(\varepsilon_{t-1}, ..., \varepsilon_{t-J}) + \varepsilon_t f(\varepsilon_{t-1}, \ldots, \varepsilon_{t-J})^2$ and both $g(.) and f(.)^2$ are nonlinear (Campbell et al., 1997). The outcomes for $f = 0$ and $g = 0$, respectively are the nonlinear Moving-Average model which is nonlinear only in mean, and the nonlinear ARCH model (Engle, 1982) which is nonlinear only in variance.

Many more nonlinear models are available in the literature, especially in the field of Financial Econometrics, and in particular those pertaining to the class of threshold autoregressions (TAR), of smooth transition autoregressions (STAR), and of the autoregressive fractionally-integrated moving average (ARFIMA) model (Granger and Joyeux, 1980; Bollerslev, 1986; Engle and