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LEO intermediary propagation as a feasible alternative to Brouwer's gravity solution ☆

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Abstract

The performance of Brouwer's gravity solution is compared with Deprit's second-order radial intermediary. Taking the main problem of the artificial satellite as the test model, that is perturbations are limited to the effects of the second zonal harmonic, it is demonstrated that the intermediary solution provides an efficient alternative for the analytical propagation of low earth orbits in a range determined by non-impact orbits with eccentricities below one tenth.

Keywords: Brouwer's theory; SGP4; Intermediaries; Elimination of the parallax

1. Introduction

Increasing needs of collision avoidance stir recent concern in reviewing SGP4 (Hoots and Roehrich, 1980), an orbit prediction model that is customarily used in the propagation of two-line element sets (see also Hoots et al. (2004), Vallado et al. (2006) and references therein). Among the reasons for this concern is the need of including uncertainties estimation in the predictions, but also the detected significant along-track errors which may be related to missing terms in the SGP4 implementation of the gravitational theory (Kelso, 2007; Easthope, 2014). These facts motivated me to search the literature for alternatives to the gravitational solution used by SGP4. I found that these alternatives do exist, can be refined to cope with more subtle short-period effects, and may provide increased efficiency in terms of precision and computation time when compared with the classical approach. The proposed alternative may be further explored to include more disturbing effects which are required for realistic propagation of space debris or orbits about asteroids.

Brouwer's celebrated closed form solution of the earth's artificial satellite problem (Brouwer, 1959) is in the roots of SGP4. In spite of the undeniable merits of Brouwer's seminal approach, or its variants for properly dealing with small eccentricities and inclinations (Lyddane, 1963; Cohen and Lyddane, 1981), proceeding by averaging is not the unique possibility in obtaining analytical solutions by general perturbations, and the use of *intermediary* orbits like useful approximate solutions of the problem of artificial satellite theory (AST) was proposed as early as in 1957 (Sterne, 1958).

AST intermediaries are commonly obtained by reorganizing the terms of the disturbing function, a simple expedient that may be preceded by the simultaneous addition and subtraction to the geopotential of some smartly-chosen supplementary terms. After reorganization, a part of the Hamiltonian that admits a separable generating function is taken as the (zero order) integrable problem whereas the rest of the disturbing function is taken as the perturbation, which may be further neglected (see Aksnes (1965), Oberti (2005) for instance). Hence, *common* intermediaries are formulated in the same variables as the original satellite problem —traditionally in spherical variables.

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It is habitually accepted that useful intermediary solutions should retain all the first-order secular and long-period effects of the artificial satellite problem and as much as possible of the short-period effects (cf. Garfinkel and Aksnes (1970)). Because of that, neither the Keplerian Hamiltonian nor the equatorial main problem[1] are considered suitable intermediaries in spite of their integrability (Jezewski, 1983). Including some of the second order secular effects of the geopotential in the intermediary is highly desirable too, and hence solutions devised by Vinti (1959, 1961, 1966) or equivalent ones based on the generalized problem of two fixed centers (see Aksenov et al. (1962), Lukyanov et al. (2005) and references therein) are considered remarkable achievements.[2]

Eventually, the efforts of Cid and Lahulla (1969) in obtaining a competitive alternative to Brouwer's solution produced a major breakthrough in the search for efficient intermediaries. Proceeding in polar-nodal variables, the usual polar ones complemented with the argument of the ascending node and its conjugate momentum, Cid and Lahulla showed that, contrary to the two canonical transformations required in Brouwer's approach, a single contact transformation is enough for removing the argument of the latitude from the main problem Hamiltonian, which after that, and in view of the cyclic character of the argument of the node, results to be integrable.

Up to the first order, Cid and Lahulla's Hamiltonian is formally equal to the radial —and, therefore, integrable— part of the main problem Hamiltonian, in this way highlighting its relationship with common intermediaries, on one hand, and disclosing the important role played by polar-nodal variables in the search for them, on the other. Besides, it incorporates all the first-order secular effects of the main problem (Deprit and Ferrer, 1987) as well as the short-period terms that affect the radial distance, thus fulfilling the traditional requisites for acceptable main problem intermediaries. Furthermore, Cid and Lahulla's solution also takes into account the remaining first-order periodic terms of the main problem, which are recovered by means of a contact transformation. All these facts inspired Deprit, who introduced the concept of *natural* intermediaries, which are integrable after a contact transformation, and demonstrated that most of the common intermediaries can be "naturalized" by finding the transformation that turns the main problem into the intermediary (Deprit, 1981). Remarkably, Deprit also showed how these contact transformations may be extended to higher orders, and proposed his own radial intermediary which, as opposite to other existing intermediaries, does not rely on the evaluation of elliptic functions. Later efforts based on Deprit's approach, showed how the generation of intermediaries of AST can be systematized (Ferrándiz and Floría, 1991).

In spite of the efforts in improving the intermediaries' performance by including second order effects of the gravity potential (Garfinkel, 1959; Aksnes, 1967; Alfriend et al., 1977; Deprit, 1981; Cid et al., 1986; Deprit and Ferrer, 1989), increasing requirements on satellite orbit prediction accuracy soon led to a decline in interest in intermediaries, to favor instead analytical and semi-analytical theories based on averaging. Still, the use of intermediary orbits of AST is enjoying a revival these days, and new applications of intermediary-based solutions had been recently encouraged for different purposes, as for predicting long-term bounded relative motion (Lara and Gurfil, 2012) or like a suitable choice for onboard orbit propagation as opposite to the usual numerical integration (Gurfil and Lara, 2014).

Here, the main problem of AST is taken as a test model to demonstrate that the use of intermediary orbits may provide an efficient alternative to Brouwer's gravity solution for the propagation of low earth orbits (LEO) in the short time intervals required by usual catalogue maintenance. Indeed, while natural intermediary closed-form solutions are unquestionably of higher complexity than Brouwer's secular terms and, in consequence, their evaluation becomes much more computationally costly, they only rely on one simplification transformation whose terms are definitely simpler than the corresponding Fourier series required by Brouwer's double averaging approach. Besides, AST intermediaries do not suffer from mathematical singularities at the critical inclination (see reviews in Coffey et al. (1986), Jupp (1988), Lara (2014)) and hence do not need to rely upon the functional patches on which higher-order analytical theories by averaging unavoidably depend —cf. Section 7 of Coffey et al. (1996) or Appendix A.F. in Hoots et al. (2004).[3]

On the other hand, efforts in solving higher-order intermediaries by separation of variables have been unsuccessful, in spite of their integrable character. This shortcoming imposes general perturbations algorithms to progress with an additional transformation in order to achieve the analytical solution of the intermediary to higher orders (Aksnes, 1967; Deprit, 1981; Deprit and Richardson, 1982). However, this extra transformation may well be avoided in the case of LEO, an instance in which orbital eccentricity is small. In particular, this is the case of Deprit's radial intermediary, or DRI in short (Deprit, 1981). For the lower-eccentricity orbits, which are a vast majority in a space catalogue of earth orbiting objects, DRI can be solved

[1] We call the *equatorial main problem* to the perturbed Keplerian motion of a satellite in the main problem assumptions that is constrained to lie on the earth's equatorial plane.

[2] Vinti found a particular choice for the coefficients of the zonal potential that makes it separable in oblate spheroidal coordinates. Vinti's analytical solution closely applies to earth's orbits where the zonal coefficient of degree 4 almost matches his choice. The equivalence between Vinti's model and the problem of two fixed centers was later recognized.

[3] It must be emphasized that neither analytical theories based on a double averaging nor existing intermediaries can deal properly with second-order perigee effects, which are crucial in the study of frozen-perigee orbits. This special class of orbits is usually approached either from a semi-analytical perspective or a purely numerical one (Cutting et al., 1978; Broucke, 1994; Coffey et al., 1994; Lara, 1999, 2008).