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A study of the main resonances outside the geostationary ring

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Abstract

We investigate the dynamics of satellites and space debris in external resonances, namely in the region outside the geostationary ring. Precisely, we focus on the 1:2, 1:3, 2:3 resonances, which are located at about 66 931.4 km, 87 705.0 km, 55 250.7 km, respectively. Some of these resonances have been already exploited in space missions, like XMM-Newton and Integral.

Our study is mainly based on a Hamiltonian approach, which allows us to get fast and reliable information on the dynamics in the resonant regions. Significative results are obtained even by considering just the effect of the geopotential in the Hamiltonian formulation. For objects (typically space debris) with high area-to-mass ratio the Hamiltonian includes also the effect of the solar radiation pressure. In addition, we perform a comparison with the numerical integration in Cartesian variables, including the geopotential, the gravitational attraction of Sun and Moon, and the solar radiation pressure.

We implement some simple mathematical tools that allows us to get information on the terms which are dominant in the Fourier series expansion of the Hamiltonian around a given resonance, on the amplitude of the resonant islands and on the location of the equilibrium points. We also compute the Fast Lyapunov Indicators, which provide a cartography of the resonant regions, yielding the main dynamical features associated to the external resonances. We apply these techniques to analyze the 1:2, 1:3, 2:3 resonances; we consider also the case of objects with large area-to-mass ratio and we provide an application to the case studies given by XMM-Newton and Integral.

Keywords: Space debris; Resonance; High area-to-mass ratio

1. Introduction

Resonances play a major rôle in the dynamics of satellites around the Earth; indeed, most of the satellites in MEO and GEO regions[1] are positioned in correspondence with the 2:1 and 1:1 gravitational resonance (see, e.g., Ely and Howell, 1997; Hubaux and Lemaître, 2013; Klinkrad, 2006; Rossi, 2008; Rossi and Valsecchi, 2006; Valk et al., 2009a,b). Being in such resonances implies that the satellite makes two orbits or one orbit during one rotation of the Earth around its spin-axis. Other resonances might be important as well, and the aim of this work is to investigate some external resonances, precisely the 1:2, 1:3, 2:3 resonances, whose distances from the center of the Earth are, respectively, about 66 931.4 km, 87 705.0 km, 55 250.7 km. A remarkable fact is that space missions have already used these resonances; in particular, Integral (International Gamma-Ray Astrophysics Laboratory) has a semimajor axis corresponding to the 1:3 resonance, while the semimajor axis of XMM-Newton (X-ray Multi-Mirror Mission) is at the 1:2 resonance. We claim that such resonances can be particularly useful when

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[1] MEO and GEO are acronyms for Medium–Earth–Orbit and Geostationary–Earth–Orbit with altitude, respectively, between 2 000 and 30 000 km for MEO and larger than 30 000 km for GEO.

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dealing with space debris; indeed, the resonant dynamics can be exploited to move space debris in safe regions, either placing them in the stable equilibria, which prevent chaotic variations of semimajor axis, or moving the debris along the chaotic invariant manifold associated to the hyperbolic equilibria.

In order to perform such investigation, we make use of the Hamiltonian formalism, allowing us to make a *fast* and *accurate* description of the dynamics. In particular, we introduce a model including a suitable expansion of the geopotential in spherical harmonics. The expansion is limited to a small number of terms, which are chosen by carefully evaluating which of them are the most significative ones in specific regions of the orbital parameter space. In this way, we reduce very much the computational time, although the model still retains the essential features of the dynamics. For objects with high area-to-mass ratio we include also a suitable expansion of the contribution due to the solar radiation pressure. Also in this case we use the Hamiltonian approach.

This strategy allows us to obtain several information on the dynamics within a very short computational time, most notably the location of the equilibrium points, their dependence on the orbital parameters, the size of the resonant regions. Further information are obtained by making a cartography (Galeş, 2012) of the different regions by means of the computation of the Fast Lyapunov Indicators (Froeschlé et al., 1997, see also Celletti, 2010). In this way we obtain very detailed maps, showing the long term evolution of the semi-major axis, although within the Hamiltonian approach we limit the cartography to a model including just the geopotential and, possibly, the solar radiation pressure. As a consequence, to validate our results we make a comparison with the Cartesian equations of motion by computing maps which include also the effects of Sun and Moon. Attention must be paid in comparing the results and, precisely, while integrating the Cartesian equations of motion, we need to transform from osculating to mean orbital elements.

We also provide an application of our technique to two sample cases: XMM-Newton, which is related to the 1:2 resonance, and Integral, related to the 1:3 resonance. These space missions are characterized by a large eccentricity; henceforth, a dedicated expansion of the geopotential is necessary, in order to include terms which are relevant at high eccentricities. Again, we use the Cartesian approach to validate the results obtained through the Hamiltonian formalism, although we find that in the case of XMM-Newton the effect of the Moon shapes the resonant islands and should be included in the overall discussion to obtain an accurate description of the dynamics.

This paper is organized as follows. In Section 2 we introduce the Cartesian equations of motion, including the geopotential, the influence of Sun and Moon, and the solar radiation pressure. In Section 3 we introduce the Hamiltonian of the geopotential and we provide explicit expressions for the secular and resonant expansions of the Hamiltonian. In Section 4 we make a qualitative analysis of the resonant regions, by reducing the study to a very limited number of terms of the expansion and by using the pendulum-like structure of the resonant zone to compute the amplitude of the resonant islands. A cartography based on the computation of the Fast Lyapunov Indicators is given in Section 5, while the case of large area-to-mass ratio is investigated in Section 6. The cases of XMM-Newton and Integral are studied in Section 7, where the expansions of the geopotential have been extended to encompass the case of large eccentricities. Some conclusions and perspectives are presented in Section 8.

2. Cartesian and Hamiltonian equations of motion

We consider a small body, say S, moving in the gravitational field of the Earth, which we assume to be oblate, and subject to the influence of Sun, Moon and to the effect of the solar radiation pressure (hereafter SRP). We assume that the body is so small, that its influence on Earth, Sun and Moon can be neglected. Let $\mathbf{r} = (x, y, z)$ be the radius vector of S in a geocentric quasi-inertial fixed frame.

The corresponding equations of motion are the sum of the equations describing the Earth's gravitational influence, the oblateness effect, the solar and lunar attraction, and the SRP. Let m_S, m_M be the masses of Sun and Moon, $\mathbf{r}_S, \mathbf{r}_M$ the position vectors of Sun and Moon (whose explicit expressions are given, e.g., in Montenbruck and Gill, 2000), $\mathcal{G}$ the gravitational constant. The equations of motion of S can be written as

$$\begin{aligned}\ddot{\mathbf{r}} = {} & R_3(-\theta)\nabla V(\mathbf{r}) - \mathcal{G}m_S\left(\frac{\mathbf{r}-\mathbf{r}_S}{|\mathbf{r}-\mathbf{r}_S|^3} + \frac{\mathbf{r}_S}{|\mathbf{r}_S|^3}\right) \\ & - \mathcal{G}m_M\left(\frac{\mathbf{r}-\mathbf{r}_M}{|\mathbf{r}-\mathbf{r}_M|^3} + \frac{\mathbf{r}_M}{|\mathbf{r}_M|^3}\right) \\ & + C_r P_r a_S^2 \left(\frac{A}{m}\right) \frac{\mathbf{r}-\mathbf{r}_S}{|\mathbf{r}-\mathbf{r}_S|^3}, \qquad (2.1)\end{aligned}$$

where R_3 is the rotation matrix about the Earth's polar axis, θ is the sidereal time, ∇ is the gradient in the synodic frame, $V(\mathbf{r})$ is the force function due to the attraction of the Earth:

$$V(\mathbf{r}) = \mathcal{G}\int_{V_E} \frac{\rho(\mathbf{r}_p)}{|\mathbf{r}-\mathbf{r}_p|}\, dV_E,$$

where $\rho(\mathbf{r}_p)$ is the density at some point $\mathbf{r}_p$ in the Earth and V_E is the volume of the Earth. The second and third terms in (2.1) model the attraction of Sun and Moon, respectively. The last term in (2.1) models the SRP and depends on the adimensional reflectivity coefficient C_r (fixed to 1 in this paper), the radiation pressure $P_r = 4.56 \cdot 10^{-6}\ \mathrm{N/m^2}$ for a body located at $a_S = 1$ AU, the area-to-mass ratio A/m with A the cross-section of S and m its mass.

We proceed now to write in Hamiltonian formulation the term in (2.1) corresponding to the geopotential. To this end, we introduce the action-angle Delaunay variables, denoted as $(L, G, H, M, \omega, \Omega)$, where $L = \sqrt{\mu_E a}$, $G =$