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ScienceDirect

Advances in Space Research 57 (2016) 153–165

Advances in Space Research
(a COSPAR publication)

www.elsevier.com/locate/asr

# Moon's influence on the plane variation of circular orbits

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Received 15 May 2015; received in revised form 17 September 2015; accepted 21 September 2015
Available online 30 September 2015

**Abstract**

The long-term orbital plane motion of a satellite in circular orbit has been investigated, taking into account the influence of the Earth's oblateness and luni-solar perturbations, to determine solutions characterised by initial conditions (in terms of Right Ascension of the Ascending Node and orbit inclination) which guarantee quasi-frozen orbital planes. The Moon's influence has been investigated considering the real motion of the lunar pole around the ecliptic pole, also evaluating the differences with respect to the simplified case in which these poles are considered as coincident at 23.445 deg of inclination. After having studied in detail the case of geosynchronous orbits, where at low inclination differences in the order of ±1.5 deg have been found for the optimal initial inclination with respect to the above-mentioned simplified case, the analysis has been extended to the range of altitude 20,000–60,000 km. The study has highlighted how the optimal initial conditions are strictly correlated to the launch date and how the differences between the real and the simplified cases increase with satellite altitude.


*Keywords:* Luni-solar perturbation; Frozen orbital plane; Geosynchronous orbit

## 1. Introduction

For a satellite, the possibility of remaining on a quasi-steady orbital plane represents an important property and, in the case of Mid and High Earth Orbit (MEO and HEO), where different typologies of navigation, telecommunication and remote sensing satellites operate, this condition is usually gained through a large consumption of propellant aimed at compensating the orbital perturbations, mainly due to the Earth's oblateness and luni-solar gravitational effects. In particular, for the Geostationary Earth Orbit (GEO), as is well-known, it is necessary to plan and perform expensive North–South station keeping manoeuvres, needed to maintain the satellites within their own operational boxes. These manoeuvres, which have to be periodically performed during the entire mission, lead to velocity variations ranging from 40 to 50 m/s per year (depending on the Moon's position), thus limiting the duration of the operational life of the satellites (Soop, 1983; Kechichian, 1997). In this regard, many proposals, based on optimisation techniques (e.g. Slavinskas et al., 1988; Romero et al., 2007), low-thrust engines (e.g. Circi, 2003) and solar sails (Circi, 2005) have been presented to reduce the propellant consumption associated with these GEO corrective manoeuvres.

In general, another possibility to limit the mission costs lies in the investigation of the dynamics of satellites moving under the influence of the above-mentioned perturbative effects. To this purpose, an extensive range of studies have been carried out on the third-body perturbation (only a few examples are reported here: Kaula, 1962; Lidov, 1963; Ash, 1976; Hughes, 1980; Šidlichovský, 1983; Lane, 1989; Breiter, 2001; Broucke, 2003; Gurfil et al., 2007; Belyanin and Gurfil, 2009) and some of these have focused on the possibility of obtaining orbits characterised by a

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http://dx.doi.org/10.1016/j.asr.2015.09.029

compensation of the out of plane perturbative effects (Allan and Cook, 1964; Tremaine et al., 2009; Ulivieri et al., 2013), demonstrating the feasibility of achieving this condition only at particular values of orbit inclination and proposing the exploitation of this condition for satellite applications (Friesen et al., 1992, 1993; Rosengren et al., 2014; Circi et al. 2015).

In particular, in Allan and Cook (1964), under the simplifying hypothesis that the lunar pole remains always coincident with the ecliptic pole, the long-term dynamics of the orbital plane has been investigated in detail and the initial conditions able to guarantee a frozen condition for the orbital plane have been determined.

However, as is well-known, in the real case the lunar pole accomplishes a precession motion around the ecliptic pole (and never coincides with it). Therefore, in order to determine the real motion of satellite orbital plane, it is necessary to consider the influence of the Moon's plane motion. To this end, extending the results found in Allan and Cook (1964), the effects of the Moon have been analysed here and, taking into consideration the Earth's oblateness and the long-term luni-solar perturbations, the initial conditions on the satellite orbital pole, which are associated with minimum orbital plane variation, have been determined in the range of altitude 20,000–60,000 km.

The paper is organised as follows: in Section 2, according to the theory presented in Allan and Cook (1964), the main results related to the simplified case of lunar pole coincident with the ecliptic pole have been reported. In Section 3 the analysis has been extended to the real case of mobile lunar pole and a quantification of the influence of this motion has been performed. In Section 4 the determination of the initial conditions on the satellite orbital pole able to guarantee the minimum variation of the orbital plane has been carried out; in particular, after having presented and described the obtained results for geosynchronous orbits, the analysis has been carried out as a function of the orbit altitude.

## 2. Motion of the orbital pole and equilibrium positions

As known, the perturbative effects related to the presence of a perturbing third body produce, in the motion of a satellite around a planet (primary body), both short- and long-term variations on the orbit elements of the satellite. The short-terms effects are function of the instantaneous position of the third body and consist of oscillations, which occur around the corresponding long-term variations, having a periodicity equal to the period of the orbit that the third body describes with respect to the primary (in the case of circular orbit for both satellite and perturbing body this periodicity reduces to half period in the gravity-gradient approximation). In the case of Earth satellites the short-term oscillations due to disturbing effects of the Sun and Moon are very small if compared with the corresponding long-term variations and therefore the temporal evolution of the satellite orbit can be well tracked by observing only the long-term variations. On the other hand, according to their definition, these long-term variations can be highlighted (eliminating the short-term ones) by performing a double average of the third body disturbing potential on the positions of satellite and perturbing body with respect to the planet. Thus, considering a second order approximation on the traditional Legendre polynomial expansion of the disturbing potential (gravity-gradient approximation) and executing the necessary average mathematical operations, the following expression for the disturbing potential, normalised with respect to $na^2$ ($n$ is the satellite mean motion, $a$ is the semi-major axis of satellite orbit), can be retrieved (Allan and Cook, 1964):

$$U^* = \omega_0\left[\frac{1}{2}(1-e^2)^{-5/2}(\mathbf{h}^*\cdot\hat{Z}_0)^2 - \frac{1}{6}(1-e^2)^{-3/2}\right] + \sum_{i=1}^{N}\omega_i\left\{\frac{1}{2}(\mathbf{h}^*\cdot\hat{Z}_i)^2 + e^2 - \frac{5}{2}(\mathbf{e}\cdot\hat{Z}_i)^2\right\} \tag{1}$$

In Eq. (1), which also includes the perturbative effect related to the planetary oblateness, the following parameters have been introduced: $\omega_0 = \frac{3}{2}\frac{J_2 n R_P^2}{a^2}$, where $J_2$ is the first zonal harmonic of the planetary potential expansion and $R_P$ is the planetary mean equatorial radius; $\omega_i = \frac{3}{4}\frac{\mu_i}{n a_i^3 (1-e_i^2)^{3/2}}$, where $\mu_i$ is the third-body planetary constant while $a_i$ and $e_i$ are, respectively, the semi-major axis and the eccentricity of the orbit described by the $i$th perturbing body with respect to the planet ($N$ = number of perturbing bodies); $\mathbf{e} = e\hat{\mathbf{e}}$, the eccentricity vector of satellite orbit ($\hat{\mathbf{e}}$ is the unit vector); $\mathbf{h}^* = \sqrt{1-e^2}\hat{\mathbf{K}}$, the angular momentum of satellite orbit, normalised with respect to $na^2$, with $\hat{\mathbf{K}}$ the unit vector perpendicular to the orbital plane (also called orbital pole); $\hat{\mathbf{Z}}_0$, the polar axis of the planet; $\hat{\mathbf{Z}}_i$, the unit vector perpendicular to the plane of the orbit described by the $i$th perturbing body with respect to the planet.

As is well known, by replacing Eq. (1) in the Lagrange planetary equations, the corresponding long-term temporal variations of the satellite orbit elements can be retrieved. To this purpose, given that Eq. (1) is also expressed as a function of vectorial term $\mathbf{h}^*$, a useful form of the Lagrange planetary equations that can be used here is the vectorial one found by Allan and Ward (1963). Then, following the mathematical developments offered by these authors and considering the normalised expressions for both disturbing potential ($U^*$) and angular momentum of satellite orbit ($\mathbf{h}^*$), the following compact form of the Lagrange planetary equations can be achieved:

$$\begin{aligned}\dot{\mathbf{h}}^* &= \mathbf{h}^*\times\frac{\partial U^*}{\partial \mathbf{h}^*} + \mathbf{e}\times\frac{\partial U^*}{\partial \mathbf{e}}\\ \dot{\mathbf{e}} &= \mathbf{h}^*\times\frac{\partial U^*}{\partial \mathbf{e}} + \mathbf{e}\times\frac{\partial U^*}{\partial \mathbf{h}^*}\end{aligned} \tag{2}$$