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An analysis method for control reconfigurability of linear systems

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Abstract

The reconfigurability of control systems is further researched based on the function-objective model (*FOM*). The establishment of the *FOM* has been published in the authors' former paper, solving the problem whether the system is reconfigurable without losing the desired control objective. Based on the *FOM*, the importance factor, the risk factor and the kth reconfigurability factor are proposed to evaluate the fault risks of all components and the system reconfigurability with k faults. These factors show which components should be improved and which faults cannot be tolerated. The analysis results are very useful for enhancing the fault-tolerance performances of the control systems by improving system designs. A satellite model is utilized to illustrate the proposed method.

Keywords: Fault-tolerant control (*FTC*); Function-objective model (*FOM*); Reconfigurability; Satellite

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1. Introduction

Failures in control systems of aircraft or satellite may lead to serious consequences or even disasters (Tafazoli, 2009), henceforth calling for strong fault-tolerant performances. Much effort has been done in fault-tolerant control method (*FTC*) (Zhang and Annual, 2008; Xiao et al., 2013), including fault detection and diagnosis (*FDD*) (Gao and Duan, 2014; Henry, 2013), and reconfigurable control method (Cai et al., 2008; Alwi and Edwards, 2010). Fault-tolerant control pays emphases on methods of diagnosing fault and utilizing redundancy, so as to reduce the fault effects.

However, whether a control system can be reconfigured is determined by the redundant components relevant to the failure components, i.e., whether there are enough redundant components to substitute the faulty components. If a key component fails and redundancy is not enough, the system can not be reconfigured whatever *FTC* method is adopted. To consider reconfigurability (Siddiqi, 2006; Wenjie et al., 2014) in the design stage (improving the redundancy design) is the essential way to improve the fault-tolerant performance of a control system.

Reconfigurability is closely associated with control objectives. For example (Gehin et al., 2012), three categories are meant for reconfiguration: (1) continuing system operation without intolerable loss of performance, (2) continuing system operation with reduced specifications, or (3) abandoning the mission while still avoiding disaster. It is the basic reconfigurability problem that whether certain faults can be tolerant without losing the desired control objectives.

Up to now, many works have been done (Markley et al., 2010; Servidia and Sanchez Peña, 2002; Servidia, 2010; Wang et al., 2010) for configurations of actuators or sensors to obtain better reconfigurability. Nevertheless analysis methods on the system level are more useful. To analyze the system reconfigurability qualitatively, structural analysis can estimate the final performance of a fault tolerant

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control system (Dstegör et al., 2006; Staroswiecki, 2007) and can determine the possibility of fault detectability/isolability from a component view. The generic component model (Gehin and Staroswiecki, 2008) provides a systematic tool for establishing different reconfiguration strategies from a function view qualitatively. To analyze the system reconfigurability qualitatively, the smallest second-order mode is proposed to be used to measure the system reconfigurability (Eva Wu et al., 2000). Based on a function tree, some indicators are proposed to analyzes system reconfigurability quantitatively (Liu and Wang, 2013). Each of the above methods contributes to this field and successfully solves a certain aspect of the reconfigurable control problems. However, a more general way to evaluate system reconfigurability both qualitatively and quantitatively is more urgently needed for practical systems.

In this paper, the following problems are suggested to be solved for evaluating system reconfigurability:

Problem I: Is the system reconfigurable when faults occur?
Problem II: What components should be adopted?
Problem III: Can the reconfigured system achieve the assigned control objective?
Problem IV: Which components have the highest fault risks?
Problem V: How to evaluate the system's reconfigurability?

The afore-mentioned methods can not answer all these problems. In our previous Chinese paper (Wenjie et al., 2014), the *FOM* has been proposed, and Problem I–III can be solved. In this paper, based on the *FOM*, some performance indicators are defined to evaluate component fault risks and system's reconfigurability (Solving Problem IV–V). The answers for Problem IV–V are useful for improving system designs so as to enhance reconfigurability.

The remainder of this paper is organized as the follows. Some basic definitions, including functions, objectives, and feasible sets, are introduced in Section 2. Section 3 presents four steps for formulating the *FOM*. Section 4 presents how to solve Problems I–V based on the *FOM*. Section 5 illustrates the presented method by analyzing a satellite control system, and conclusions are presented in Section 6. For the completeness of this method, the establishment of the *FOM* is introduced in this paper. Readers who have read our former paper (Wenjie et al., 2014), Sections 2, 3 and 4.1 can be neglected.

2. Theoretical background

Fig. 1 illustrates the *FTC* strategy with control allocation, where $\boldsymbol{v}$ is the virtual control, and $\boldsymbol{u}$ is the desired control vector. The control allocation distributes $\boldsymbol{v}$ to $\boldsymbol{u}$. The predefined algorithms (controller, control allocation, *FDD*), actuators, and sensors constitute the control system, where faults may occur in any of the components.

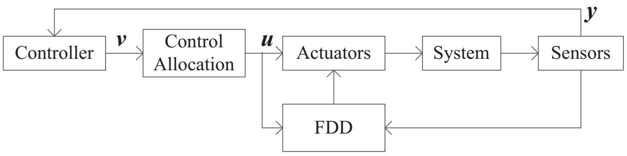

Fig. 1. Fault-tolerant control strategy.

The linear state-space model is written as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

where $A \in \mathrm{R}^{n_A \times n_A}$, $B \in \mathrm{R}^{n_A \times n_u}$, $C \in \mathrm{R}^{n_C \times n_A}$, $\boldsymbol{x} \in \mathrm{R}^{n_A}$, and $\boldsymbol{y} \in \mathrm{R}^{n_C}$.

The faulty system can be written as

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + B_f\boldsymbol{u} \\ \boldsymbol{y} = C_f\boldsymbol{x} \end{cases} \tag{2}$$

where $B_f \in \mathrm{R}^{n_A \times n_{uf}}$, $C_f \in \mathrm{R}^{n_{Cf} \times n_A}$.

The controllability criterion and observability criterion are

$$rank\left([B_f, AB_f, \ldots, A^{n_A-1}B_f]\right) = n_A \tag{3}$$

$$rank\left(\left[C_f^T, A^\top C_f^\top, \ldots, (A^T)^{n_A-1}C_f^T\right]\right) = n_A \tag{4}$$

In this paper, we assume that (1) is controllable and observable. If (2) satisfies (3) and (4), the faulty system is considered to be reconfigurable when there are no other requirements.

2.1. Definitions for the structural decompositions

The control system can be split into minimal reconfigurable units (*MRU*) using the following hierarchy: system → subsystem → … → reconfigurable unit (*RU*) → *MRU*.

An *MRU* is a minimal component of the system. An *MRU* cannot tolerate a fault by itself, and the system can only restore the corresponding functions using other *MRU*s.

An *RU* is a group of *MRUs*. For example, all of the gyros in a satellite control system (a subsystem) may be considered as an *RU*. If one gyro breaks down, the other gyros can still correctly measure the system's angular velocity if there is enough redundancy. RU_j is composed of n_p *MRUs*, defined as

$$RU_j = \sum MRU_i, \quad i = 1, \ldots, n_p \tag{5}$$

After structural decomposition, a control system including n *MRUs* is

$$S = \sum MRU_i, \quad i = 1, \ldots, n \tag{6}$$

Each *MRU* may have several states, such as totally fault, proportional fault (discussed later), or normal. We give a definition, minimal reconfigurable unit state (*MRUS*), to describe the state of the *MRU*. Take a momentum wheel (MRU_j) with four states for example: closed ($MRUSj1$),