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The effect of the Earth's oblateness on predicting the shadow conditions of a distant spacecraft: Application to a fictitious lunar explorer

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Abstract

The effect of the Earth's oblateness on predicting the shadow events of a lunar spacecraft caused by the Earth's shadow is analyzed in this study. To ensure a reliable analysis, the proven 'line-of-intersection' method is modified and directly applied to predict the shadow conditions using a spheroidal model of the Earth and a conical shadow model. Two major lunar mission phases, namely, transfer and orbiting, are considered with corresponding fictitious initial conditions, and eclipse events are predicted and the results are compared using both spherical and spheroidal Earth models. For the lunar transfer phase, for which an Earth-bound highly elliptical orbit is assumed, not only the predicted entry and exit times of an event but also its duration are found to be more strongly shifted as the apogee altitude increases; for perigee and apogee altitudes of 1000 and 380,000 km, respectively, the maximum difference in predicted duration is found to be approximately 0.76 min for a penumbra event. For the lunar orbiting phase, for which a circular orbit around the Moon at an altitude of 100 km is assumed, a prediction difference of approximately half a minute on average and approximately one minute at maximum (e.g., 0.73 min for qumbra events, 1.03 min for penumbra events and 1.32 min for 'instantaneous' full sunlight events) can occur. The results of the present analysis highlight the importance of modeling the oblate shape of the Earth when predicting the shadow events of a distant spacecraft, and they are expected to provide numerous insights for any missions involving highly elliptical orbits around the Earth or travel to the Moon.

Keywords: Spheroidal Earth shape; Shadow prediction; Eclipse event; Lunar spacecraft

1. Introduction

At present, space missions are becoming increasingly complex while needing to satisfy higher accuracy requirements at lower costs. To accommodate these trends, innovative and competitive technologies for space missions are being developed not only to reduce costs but also to increase mission effectiveness; such technologies include solar electric and solar sail propulsion systems for orbit transfer. Smaller satellites are also emerging as replacements for larger-sized satellites. In the coming decade, it is expected that many diverse missions could be accomplished, including planetary exploration, using these innovative technologies. Indeed, numerous planetary missions have already been performed using solar electric propulsion systems and have demonstrated the effectiveness of these systems in maximizing payload capacities. Moreover, the NASA Innovative Advanced Concepts (NIAC) program recently selected a small interplanetary spacecraft for further investigation to enable a new class of missions beyond low Earth orbit (LEO) (Blaney et al., 2012). In every phase of a space mission, (i.e., design, analysis and operation), shadow condition analysis plays an important

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role in tasks such as ensuring the effective utilization and sizing of solar panel arrays and batteries, deducing the appropriate thermal control and power management requirements, determining opportunities for mission access, etc. (Larson and Wertz, 1999; Wertz, 2001). Moreover, if such missions require highly precise and accurate orbit prediction, then precise shadow event prediction is of even greater importance.

There are two general categories of shadow models that are used for the prediction of shadow conditions: cylindrical and conical models (Escobal, 1976). As the name implies, a cylindrical model assumes the shadow region to be a cylindrical projection of the blocking body, whereas in a conical model, the blocking body's shadow region consists of two distinct conical projections: the umbra and penumbra regions. In the umbra region, the spacecraft experiences a total eclipse of solar irradiance, and it experiences a partial eclipse in the penumbra region. In the past, most practical shadow condition analyses have used a cylindrical shadow model with a spherical Earth shape (Cunningham, 1962; Long et al., 1989; Mullins, 1991) because this simplification of the occultation model significantly reduces the computational load (Ortiz and Rickman, 1995). However, it has been noted that this simplified model is valid only for a circular, low-attitude orbit, and it has been concluded that a conical model should be used to avoid significant terminator point calculation errors at high altitudes or in the case of highly elliptical orbits (HEOs) (Ortiz and Rickman, 1995). In fact, Woodburn (2000) has shown that ignoring the penumbra region by adopting the cylindrical shadow model results in serious errors in the numerical integration of a spacecraft's orbit. Hubaux et al. (2012) proposed an innovative symplectic integration scheme that computes shadow crossings for a spherical Earth (for both cylindrical and conical shadow models) while numerically integrating the space debris orbit using the Earth's gravitational potential, planetary and luni-solar gravitational perturbations, and direct solar radiation pressure. Later, Hubaux et al. (2013) analyzed the influence of shadowing effects on the regular or chaotic behavior of space debris orbits.

Although the conical shadow model has been used to predict the shadow conditions in numerous studies (Montenbruck and Gill, 2012; Srivastava et al., 2015a; Vallado, 2013; Wertz, 1980), the shape of the Earth is still assumed to be spherical in most of these studies to simplify the problem. In reality, however, the Earth has a flattened shape; the polar radius of the Earth is approximately 0.3% shorter than the equatorial radius. Thus, the umbra and penumbra transition times may vary with the overall eclipse duration. To describe this phenomenon, Vokrouhlicky et al. (1996) modeled the Earth as an oblate spheroid for the prediction of shadow conditions. However, the approaches used to determine the eclipse boundary conditions using the spheroidal Earth model were computationally expensive to implement in the available software. Therefore, Adhya et al. (2004) developed the simple but effective 'line-of-intersection' method to determine the eclipse states using the spheroidal Earth model. These authors also validated the algorithm's performance and showed that the accuracies of the calculated shadow crossing times, which remained within 1 s, compared well to the real flight data of LEO satellites. Recently, additional tests were conducted using real Indian Remote Sensing (IRS) LEO satellite data (Srivastava et al., 2014). Srivastava et al. (2014) reported that prediction errors on the order of several seconds are expected when the umbra and penumbra transition times are predicted using a spherical Earth model. Extensive studies on the prediction of eclipse events for an oblate-shaped blocking body have also been performed. Srivastava et al. (2015b) analyzed shadow eclipse predictions considering the effects of the atmospheric dust of Mars assuming both spherical and spheroidal shapes for Mars. Also, the shadow conditions caused by the shadow of the Moon falling onto Earth-orbiting spacecraft (i.e., spacecraft with LEOs and geostationary orbits (GEOs)) have been studied (Srivastava et al., 2015c). Several studies have confirmed that the use of a spheroidal model for the blocking body and a conical shadow model is necessary to reduce the errors on the predicted shadow transition times for precise mission analysis and operation (Adhya et al., 2004; Srivastava et al., 2014; Vokrouhlicky et al., 1996). Nevertheless, recent studies performed for Earth-orbiting spacecraft have been limited only to LEO spacecraft and have concluded that adopting a spheroidal shape model for the Earth is required only when predicting the eclipse conditions at higher altitudes, such as for a medium Earth orbit (MEO), a GEO or a planetary mission (Srivastava et al., 2014).

Similar to the predicted accuracy degradation observed between the cylindrical and conical shadow models, it can be expected that the effect of the Earth's oblateness on the prediction of precise umbra and penumbra transition times may be significant at large distances between the Earth and the spacecraft. Therefore, the primary objective of this study is to analyze the effect of the Earth's oblateness on the predictions of shadow events for distant spacecraft and thus to provide not only rough estimates of the expected errors but also practical insights into the design of such missions. In this case, a lunar spacecraft is the ideal candidate for investigating the effect of the Earth's oblateness on predictions of the shadow conditions of a distant spacecraft. A lunar spacecraft will establish an HEO around the Earth during the transfer phase; in fact, a direct transfer strategy may also be used, but the current analysis will consider only an Earth-bound transfer strategy with an HEO for the purpose of focusing on eclipse event predictions. After being captured by the Moon, the spacecraft will continuously orbit at the Earth–Moon distance. Therefore, all orbits of such a lunar spacecraft will lie within the Earth's shadow, and these orbits will be strongly affected by the Earth's oblateness. In fact, a similar analysis was performed for spacecraft at the