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Finite gyroradius corrections in the theory of perpendicular diffusion 2. Strong velocity diffusion

A. Shalchi

Department of Physics and Astronomy, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

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Abstract

The current paper is a sequel to an article where we have started to incorporate finite gyroradius effects in the theory of perpendicular diffusion of energetic particles interacting with turbulent magnetic fields. In the previous paper we have focused on the case that velocity diffusion is suppressed and we derived corrections to the perpendicular diffusion coefficient. In the current article we focus on the limit of strong non-linear velocity diffusion. If finite gyroradius effects are not present, we derive the well-known limit where the perpendicular diffusion coefficient is directly proportional to the parallel diffusion coefficient. As in the previous paper, we employ different turbulence models as examples, namely the slab model, noisy slab turbulence, and the two-dimensional model. We show that finite gyroradius effects reduce the perpendicular mean free path in all considered cases except for the slab model where such effects do not occur.

Keywords: Magnetic fields; Turbulence; Energetic particles

1. Introduction

A fundamental problem in plasma physics, space science, and astrophysics is the motion of energetic particles such as cosmic rays through a turbulent magnetized plasma. Due to the interaction with such turbulent fields, the particles experience diffusion along and across the large scale magnetic field (see, e.g., Schlickeiser (2002), Spatschek (2008), and Shalchi (2009) for reviews). In Shalchi (2015b) we have started to incorporate the effect of a finite gyroradius and explored the consequences for the perpendicular diffusion coefficient. In the following we refer to this paper as *Paper I*. All details concerning the physics of transport theory and applications can be found there. In what follows, we briefly summarize the most important points of *Paper I*.

In linear and non-linear analytical treatments of perpendicular diffusion, one usually starts with the following equation of motion (see, e.g., Matthaeus et al. (2003))

$$V_x(t) = a v_z(t) \delta B_x[\vec{x}(t)]/B_0. \tag{1}$$

Here we have used the mean magnetic field B_0, the turbulent field δB_x at the position of the charged particle $\vec{x}(t)$, the parallel component of the particle velocity vector v_z, and the x-component of the particle's guiding center velocity vector V_x. Critical here is the parameter a which is usually assumed to be a number between $a^2 = 1/3$ and $a^2 = 1$ (see, e.g., Matthaeus et al. (2003) and Tautz and Shalchi (2011)). As in *Paper I* we understand the latter parameter as finite gyroradius correction. This means that if such corrections do not occur, we have by definition $a^2 = 1$.

In equations of motion, such as Eq. (1), one finds the vector $\vec{x}(t)$ which indicates the position of the charged energetic particle. Instead of the particle coordinates (x, y, z) one can use the coordinates (see, e.g., Schlickeiser (2002))

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E-mail address: andreasm4@yahoo.com

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$$X = x + \frac{v_y}{\Omega}, \quad Y = y - \frac{v_x}{\Omega}, \quad \text{and} \quad Z = z \tag{2}$$

which are usually called the *guiding center coordinates* (X, Y, Z). We like to point out that the latter coordinates only correspond to the actual position of the guiding center in the unperturbed case ($\delta\vec{B} = 0$) because only then the guiding center is well-defined. For the velocity $\vec{v}$ of the particle we use spherical coordinates

$$\begin{aligned} v_x &= v\sqrt{1-\mu^2}\cos\Phi \\ v_y &= v\sqrt{1-\mu^2}\sin\Phi \\ v_z &= v\mu \end{aligned} \tag{3}$$

with the particle speed v, the pitch-angle cosine μ, and the gyrophase Φ. Usually, in transport theory, we describe the particle motion in a six-dimensional phase space with the coordinates (X, Y, Z, v, μ, Φ).

In analytical diffusion theories, as well as in test-particle simulations, one has to specify the turbulent magnetic fields. A fundamental quantity to do so is the so-called magnetic correlation tensor whose components are defined via

$$P_{mn}\left(\vec{k}\right) := \left\langle \delta B_m\left(\vec{k}\right)\delta B_n^*\left(\vec{k}\right)\right\rangle \tag{4}$$

where $\vec{k}$ is the wavevector. The latter tensor has to be specified if a diffusion coefficient is calculated. In the current paper we consider the same examples as in *Paper I*, namely:

- The slab model.
- A noisy slab model.
- Two-dimensional turbulence.

In *Paper I* we have used quasi-linear theory and non-linear diffusion theory for the case of suppressed velocity diffusion in order to compute the perpendicular diffusion coefficient. In the current article we use non-linear theory for the case of strong velocity diffusion and compute the perpendicular diffusion coefficient by taking into account the finite gyroradius of the particle. In Section 2 we derive fundamental equations and in Section 3 we consider the zero gyroradius limit. In Section 4 we explore perpendicular diffusion for the turbulence models listed above. We conclude and summarize in Section 5.

2. Non-linear diffusion theory

In the following we use ideas of non-linear diffusion theory for the special case of strong velocity diffusion. This type of non-linear approach is based on the work of Shalchi et al. (2004) and Shalchi (2010).

2.1. The non-linear approach

As explained in *Paper I*, the Fokker–Planck coefficient of perpendicular diffusion is calculated via the TGK formulation (see Taylor (1922), Green (1951), and Kubo (1957))

$$\begin{aligned} D_\perp &= \Re\int_0^\infty dt\,\langle V_x(t)V_x(0)\rangle \\ &= \frac{1}{B_0^2}\Re\int_0^\infty dt\,\langle v_z(t)v_z(0)\delta B_x(t)\delta B_x(0)\rangle \end{aligned} \tag{5}$$

where we have employed Eq. (1) and set $a^2 = 1$ therein. Furthermore, we have used the notation $\delta B_x(t) = \delta B_x[\vec{x}(t)]$.

If the parameter $D_\perp$ is known, one can compute the perpendicular diffusion coefficient by calculating the pitch-angle average of the corresponding Fokker–Planck coefficient (see, e.g., Schlickeiser (2002))

$$\kappa_\perp = \frac{1}{2}\int_{-1}^{+1} d\mu\, D_\perp(\mu). \tag{6}$$

Alternatively, the perpendicular mean free path can be considered, which is defined via $\lambda_\perp = 3\kappa_\perp/v$.

The average operator $\langle\ldots\rangle$ used in Eq. (5) contains the average over different particle properties. After deriving the Fokker–Planck coefficient $D_\perp(\mu)$ the spatial diffusion coefficient $\kappa_\perp$ is obtained by computing the pitch-angle average via Eq. (6). In non-linear diffusion theories it is often more convenient to combine both average operations and to define

$$\begin{aligned} \langle\langle A\rangle\rangle &:= \frac{1}{2}\int_{-1}^{+1} d\mu\,\langle A\rangle \\ &= \frac{1}{(4\pi)^2}\int_{-1}^{+1} d\mu\int_{-1}^{+1} d\mu_0\int_0^{2\pi} d\Phi\int_0^{2\pi} d\Phi_0 \\ &\quad\times\int d^3X\, A\left(\vec{X},\Phi,\mu,t\right)f\left(\vec{X},\Phi,\mu,t\right) \end{aligned} \tag{7}$$

with the phase-space distribution function $f(\vec{X},\Phi,\mu,t)$. In Eq. (7) we have used the gyrophase Φ, the initial gyrophase Φ_0, and the initial pitch-angle cosine μ_0. Therewith, the perpendicular diffusion coefficient can be written as

$$\kappa_\perp = \frac{1}{B_0^2}\Re\int_0^\infty dt\,\langle\langle v_z(t)v_z(0)\delta B_x(t)\delta B_x(0)\rangle\rangle. \tag{8}$$

Obviously, the critical quantity in the theory of perpendicular diffusion is the 4th order correlation function

$$C(t) := \langle\langle v_z(t)v_z(0)\delta B_x(t)\delta B_x(0)\rangle\rangle. \tag{9}$$

In the following we aim to compute the perpendicular diffusion coefficient based on the formulation explained above. The turbulent magnetic field in Eq. (9) can be replaced by a Fourier representation leading to

$$C(t) \approx \int d^3k\,\left\langle\left\langle \delta B_m\left(\vec{k}\right)\delta B_n^*\left(\vec{k}\right)v_z(t)v_z(0)e^{i\vec{k}\cdot[\vec{x}(t)-\vec{x}(0)]}\right\rangle\right\rangle. \tag{10}$$

As in *Paper I*, we employ the so-called *random-phase approximation*

$$C(t) \approx \int d^3k\, P_{xx}(\vec{k})\left\langle\left\langle v_z(t)v_z(0)e^{i\vec{k}\cdot[\vec{x}(t)-\vec{x}(0)]}\right\rangle\right\rangle \tag{11}$$