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Approaches to relativistic positioning around Earth and error estimations

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Abstract

In the context of relativistic positioning, the coordinates of a given user may be calculated by using suitable information broadcast by a 4-tuple of satellites. Our 4-tuples belong to the Galileo constellation. Recently, we estimated the positioning errors due to uncertainties in the satellite world lines (U-errors). A distribution of U-errors was obtained, at various times, in a set of points covering a large region surrounding Earth. Here, the positioning errors associated to the simplifying assumption that photons move in Minkowski space–time (S-errors) are estimated and compared with the U-errors. Both errors have been calculated for the same points and times to make comparisons possible. For a certain realistic modeling of the world line uncertainties, the estimated S-errors have proved to be smaller than the U-errors, which shows that the approach based on the assumption that the Earth's gravitational field produces negligible effects on photons may be used in a large region surrounding Earth. The applicability of this approach – which simplifies numerical calculations – to positioning problems, and the usefulness of our S-error maps, are pointed out. A better approach, based on the assumption that photons move in the Schwarzschild space–time governed by an idealized Earth, is also analyzed. More accurate descriptions of photon propagation involving non symmetric space–time structures are not necessary for ordinary positioning and spacecraft navigation around Earth.

Keywords: General relativity; Relativistic positioning systems; Global navigation satellite systems; Methods: numerical

1. Introduction

This paper focuses on the estimation of positioning errors in some relativistic positioning systems (RPS). Realizations of RPS must be based on general relativity (GR); more specifically, the following theoretical scheme is to be implemented:

(i) Space–time is governed by an energy distribution including Earth, Sun and other sources. It is described by a metric, which must be written in terms of certain coordinates, y^α, being appropriate to deal with positioning.
(ii) In the absence of non gravitational forces, satellites and photons move in the aforementioned space–time as test particles; namely, following geodesics. In particular, satellites follow time-like geodesics which may be parametrized by the proper times.
(iii) Once the metric and the world lines of satellites and photons are known, any user may calculate its y^α coordinates by using the proper times broadcast by four satellites, the so-called emission coordinates τ^A.
(iv) These emission coordinates and the satellite world lines – parametrized by the proper times – set the satellite coordinates at emission, which are also

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the initial photon coordinates; namely, the coordinates at the starting point of the photons broadcasting information.

(v) Among the photon geodesics it is possible to find four intersecting ones (one per satellite), and the coordinates y^{α} of the resulting intersection event define the user position.

Any approach to RPS based on these general ideas requires additional assumptions and approximations.

In our opinion, the estimation of positioning errors is more systematic in the RPS context. Instead of applying a correction for each effect leading to errors, let us systematically proceed taking into account the following points:

(1) We define *nominal satellite world lines*, which are appropriate time-like geodesics of the RPS space–time. These geodesics would be the true satellite world lines in the absence of perturbations.
(2) In a given RPS, there are non gravitational perturbations such as solar winds, radiation pressure, and so on, as well as gravitational perturbations due to the energy sources which have not been taken into account to fix the space–time structure. All these perturbations would produce growing deviations with respect to the nominal world lines.
(3) These deviations will not be estimated but controlled; namely, the satellite world lines will be corrected as soon as the amplitude of their deviations reaches a certain limit value.
(4) The amplitude evolution and the statistical character of the deviations will be determined from the analysis of many data, which may be obtained by measuring deviations over an extended period (many satellite periods).
(5) Once nominal world lines and statistical realizations of the deviations are available, the nominal world lines will be used to calculate the user position, and the deviations to estimate errors. These errors will be hereafter called U-errors since they are due to uncertainties in the satellite world lines.
(6) The best RPS would be obtained taking into account all the sources contributing to the gravitational field. In such a case, only the non gravitational forces would produce deviations with respect to the nominal world lines and, consequently, less corrections of the satellite motions would be necessary to maintain the deviations smaller than the chosen limit amplitude.
(7) There are other positioning errors associated to the description of photon propagation (from the satellites to the user). These errors arise when some sources of the gravitational field are neglected and, consequently, the metric and the photon null geodesics are not fully accurate.

Two approaches to relativistic positioning are considered in this paper, in both cases we look for the positioning coordinates and their errors. These RPS are designed by assuming that the space–time has the Schwarzschild metric, which corresponds to an ideal isolated static spherically symmetric Earth. Schwarzschild space–time is asymptotically Minkowskian and, consequently, once the approach based on Schwarzschild metric is assumed, one can say that, from a theoretical (physical) point of view, there are inertial (quasi inertial) systems of reference. The origin of these references is located in the Earth's center and the spatial axes are arbitrary. The simplest of these two approaches, hereafter called the 0-order RPS, is based on the following assumptions: (a) satellite world lines are time-like geodesics of the Schwarzschild space–time (hereafter S-ST), and (b) photons follow null geodesics in the Minkowski space–time (M-ST) asymptotic to the Schwarzschild space–time. In a more accurate approach, hereafter called the 1-order RPS, both satellites and photons move in S-ST. Here, the accuracy of the 0-order RPS is quantitatively estimated, for the first time, in an extended region surrounding Earth. This estimation is based on the calculation of the S-errors, which are the differences between the positioning coordinates obtained in the 0 and 1-order RPS.

In Puchades and Sáez (2014), the U-errors were estimated inside a spherical region, with radius $R = 10^5$ km, centered at point E. The spherical inertial coordinates of E were assumed to be $r_E = R_{\oplus}$, $\theta_E = 60°$, and $\phi_E = 30°$, where $R_{\oplus}$ is the Earth's radius. Hence, point E is on the Earth's surface. It is an arbitrary point and results do not depend on its choice. In this paper, other positioning errors (S-errors) are estimated inside the same sphere to facilitate comparisons with the U-errors. This great region around Earth is hereafter referred to as the E-sphere.

We only consider the Galileo constellation, whose satellites are being placed in orbit by the European Space Agency. This GNSS (global navigation satellite system) has 27 satellites, which are uniformly distributed on three equally spaced orbital planes. We have numbered the satellites in such a way that numbers 1–9, 10–18, and 19–27 correspond to consecutive orbital planes. Inside any of these planes, satellites n and $n+1$ occupy successive positions. The inclination of these planes is $\alpha_{in} = 56°$ and the altitude of the circular orbits is $h = 23,222$ km; thus, the orbital period is about 14 h. See Pascual-Sánchez (2007) for details. Our nominal world lines are chosen to be Schwarzschild time-like geodesics with these circular orbits.

Let us make some comments about notation and units which will be taken into account in the whole paper. Index A labels the four satellites necessary for space–time positioning; any other Latin index runs from 1 to 3, and Greek indexes from 1 to 4. Quantities G, $M_{\oplus}$, t, and τ stand for the gravitation constant, the Earth's mass, the coordinate time, and the proper time, respectively. In our numerical codes, units are chosen in such a way that the speed of light is $c = 1$; the kilometer is the unit of distance, and times are given in units of $10^{-5}/3$ s (hereafter *code units*). In all the equations we set $c = 1$ and, finally, results (code outputs)