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Finite element analysis of an inflatable torus considering air mass structural element

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Abstract

Inflatable structures, also known as gossamer structures, are at high boom in the current space technology due to their low mass and compact size comparing to the traditional spacecraft designing. Internal pressure becomes the major source of strength and rigidity, essentially stiffen the structure. However, inflatable space based membrane structure are at high risk to the vibration disturbance due to their low structural stiffness and material damping. Hence, the vibration modes of the structure should be known to a high degree of accuracy in order to provide better control authority. In the past, most of the studies conducted on the vibration analysis of gossamer structures used inaccurate or approximate theories in modeling the internal pressure. The toroidal shaped structure is one of the important key element in space application, helps to support the reflector in space application. This paper discusses the finite-element analysis of an inflated torus. The eigen-frequencies are obtained via three-dimensional small-strain elasticity theory, based on extremum energy principle. The two finite-element model (model-1 and model-2) have cases have been generated using a commercial finite-element package. The structure model-1 with shell element and model-2 with the combination of the mass of enclosed fluid (air) added to the shell elements have been taken for the study. The model-1 is computed with present analytical approach to understand the convergence rate and the accuracy. The convergence study is made available for the symmetric modes and anti-symmetric modes about the centroidal-axis plane, meeting the eigen-frequencies of an inflatable torus with the circular cross section. The structural model-2 is introduced with air mass element and analyzed its eigen-frequency with different aspect ratio and mode shape response using in-plane and out-plane loading condition are studied.

Keywords: Modeling and simulation of torus; Structural element; Aspect ratio; Inflation pressure; Finite-element discretization

1. Introduction

Torus is a basic structural element functioning to carry the load imposed by the inflation pressure as well as take up the tensile forces created by the stretched membrane at the edges. Now days its uses are found in civil, mechanical, aircraft, marine engineering, and in space application. The toroidal shaped structural element is a key component to the inflatable space structure. An inflated toroidal structure is often used to provide structural support to the reflector in space application. For example, the application to solar reflectors or pressurized antennas requires the membrane to assume a parabolic shape of considerable accuracy; the inflation pressure must be large enough to provide a smooth surface and eliminate any wrinkles or waviness. However, a torus, although a simple structure in concept, is actually numerically very challenging. The torus must have not only the strength to carry the load imposed by the inflation pressure, but also the rigidity to resist deformation and allow the membrane to maintain its accurate shape. The torus have two curvatures in two principal direction causes coupling between bending and stretching actions of load and gives rise to a smaller deflection compared to that of beams, plates, and cylinders. This

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makes the toroidal shell capable of carrying higher loads. In view of their importance, structural static and dynamic behavior of inflated torus needs to be investigated.

In order to improve the classical solutions, some investigators (Rao and Sundararajan, 1969; Philipson, 1956; Kirkhope, 1976) have studied the effect of extension, transverse shear, and rotary inertia for torus. Timoshenko presented static analysis of an inflated torus using linear membrane theory. While this classical solution gave an acceptable stress distribution, the solutions for displacements contained singularities (Timoshenko, 1940). Jordan used a non-linear membrane theory for an inflated torus to remove the incompatibility of displacement singularities efforts and to find a more accurate analysis (Jordan, 1962). Whereas, Colboume and Flugge resolved the issue of the displacement discontinuities by considering it as a singular perturbation problem involving the so-called pre-stress parameter that depends on the internal pressure, Young's modulus, shell thickness, and the internal radius of the torus (Colboume and Flugge, 1967). Liepins presented an extensive study of the free vibration analysis of a toroidal membrane subjected to an internal pressure using finite difference method for lower frequencies only. Another important observation was that with a low aspect ratio (radius of the tube of the torus divided by the radius of the torus) and high prestress, the torus vibrates as a ring (Liepins, 1965). A few lower vibratory frequencies using the Rayleigh quotient had been predicted (Jordan, 1966). Later, the same researcher presented experimental results for the vibration testing of inflated torus of free and fixed boundary conditions (Jordan, 1967). Siagal et al. presented the closed-form solution for a general doubly curved toroidal shell with fixed boundary conditions relied strongly on very limiting simplification assumption, namely zero in plane displacement and very small aspect ratio (Saigal et al., 1986). Leigh et al. used the Finite-element code MSC/NASTRAN to model an inflated torus along with three struts, and compared the results to test data (Leigh et al., 2001; Leigh and Tinker, 2003). Lewis and Inman discussed the effect of an internal pressure including the prestress value via piezoelectric actuators of an inflatable torus without considering the follower pressure force (Lewis and Inman, 2001). Jha and Inman demonstrated the importance of geometric nonlinearity and direct action of pressure force and presented a comparison of the results from different approximate shell theories (Jha and Inman, 2004). The experimental investigation had been performed to know the dynamic effect of the inflated torus indicating the potential use of smart materials in the actuation and control of inflated structures (Griffith and Main, 2000; Park et al., 2001, 2002; Ruggiero et al., 2002). A literature review by Ruggiero describes analytical as well as experimental works on the dynamic response of inflated torus (Ruggiero et al., 2003).

The existing research work shows that the error of one-dimensional model with circular cross section of toroidal ring increases with decrease of radius ratio (radius ratio is the inverse of aspect ratio). In addition to the one-dimensional models, some researchers also applied two-dimensional plate (Bakshi and Callahan, 1966; Tang and Bert, 1987) and shell (Taniguchi and Endo, 1971; Leissa, 1973) models to analyze the mechanical behavior of a torus. Recent literature survey found that (Chidamparam and Leissa, 1993) most of the published papers are about rings with a rectangular cross section. For rings with a circular cross section, only the one-dimensional models have been developed. Mathematical models have serious limitations in their scope of applications, which are only suitable for slender or thin structural elements. As a result, the three-dimensional analysis of structural elements has long been a goal of those who work in the field. In recent years, the Ritz method has been extensively applied to the three-dimensional vibration analysis of some typical structural elements, such as beams with a circular cross section, cylindrical shells, rectangular and trapezoidal plates, prismatic columns, shell panels, triangular plates and rings with isosceles trapezoidal, and triangular cross sections, etc. (Leissa and So, 1995; So and Leissa, 1998; Leissa and Kang, 1999; Liew et al., 1998; Cheung and Zhou, 2002; Kang and Leissa, 2000). In these references, high accuracy, a small computational cost, and easy coding preparation have been shown if suitable admissible functions are selected.

In the recent two decades, with the development of digital computers and computational techniques, it has now become possible to obtain the accurate eigen-frequencies and vibration mode shapes for some structural elements. Taylor proposed a large displacement formulation of a membrane composed of three-node triangular elements based on rectangular Cartesian coordinates. Some membrane structures have a very low flexural stiffness that can support a small amount of compressive stress before buckling appears (Taylor, 2001). In order to avoid compression stresses, membranes are prestressed. Levy and Spillers (1995), Gil (2003) use a prestressed method to analyze membranes which are initially flat in the Euclidean space. An approach that includes curved prestressed membranes using a projection scheme can be found in Bletzinger and Wuchner (2001). A general formulation for membranes based on curvilinear coordinates is given by Bonet et al. (2000), Lu et al. (2001)). Now a day's these membranes are used commercially such as parachutes, automobile airbags, sails, windmills, human tissues, and long span structures.

In the present work, the finite-element tool packages are used to generate the two model of toroidal shaped. The model-1 is to be investigated considering one with shell element only and model-2 with the combination of mass of enclosed fluid (air) added to the shell elements. The analytical approach has been presented for the inflated torus (model-1) using the Ritz method with a circular cross section based on the three-dimensional elasticity theory. The free vibration analysis of an inflated torus for model-1 has been done and computed with the present analytical method and validated with the available research. The