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Minimum of PDOP and its applications in inter-satellite links (ISL) establishment of Walker-δ constellation

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Abstract

Within the next decade, there will be a number of GNSS (Global Navigation Satellite System) available, i.e. modernized GPS, Galileo, restored GLONASS, BeiDou and many other regional GNSS augmentation systems. Thus, measurement redundancies and geometry of the satellites can be improved. GDOP (Geometric Dilution of Precision) and PDOP (Position Dilution of Precision) are associated with the constellation geometry of satellites, and they are the geometrically determined factors that describe the effect of geometry on the relationship between measurement error and position error. GDOP and PDOP are often used as standards for selecting good satellites to meet the desired positioning precision. In this paper, the related conclusions of minimum of GDOP which was discussed are given, and it is used to study the minimum of PDOP for two cases that the receiver is on the earth's surface and the receiver is on satellite. The corresponding theorem and constructive solutions of minimum of PDOP are given. Then, the rationality of the ISL (inter-satellite link) establishment criteria in Walker-δ constellation is discussed by using the theory of minimum of PDOP. Finally, the minimum of PDOP is calculated when the number of satellites is 4–10, and these results are verified by using Monte Carlo method.

Keywords: Minimum of GDOP; Minimum of PDOP; Walker-δ constellation; ISL; Monte Carlo method

1. Introduction

When pseudo-range data is used for real-time applications, the DOP (Dilution of Precision) values are the measure for the geometrical strength of the observation model. It only depends on the observable configuration and observation time of the local satellites. The satellite visibility and DOP values of each system and their combinations are used as the major indices for performance evaluation (Chen et al., 2009). Different types of DOP values are distinguished, and the effect of the geometry is characterized by the parameters known as the GDOP (Geometric Dilution of Precision) and the PDOP (Position Dilution of Precision) (Psiaki, 2006). Here we only discuss the geometric DOP and the position DOP, that is, GDOP and PDOP respectively. The relative geometry of the satellites and the user receiver determines the effect of individual pseudorange errors on the accuracy of the unknown 3-dimensional user position vector. GDOP and PDOP are generally used as a criterion for selecting optimal visible satellites to improve the positioning accuracy by using GNSS measurements of pseudo-range or phase. As for GPS, the constellation's orbital geometry has been designed so that at least 5 satellites are visible in any part of the globe, and currently the average number of visible satellites is 7 or 8; Typical values of GDOP range from 2

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to 3 around the globe if there is a clear view of the sky, but GDOP values may reach or even exceed 10 if obstructions leave only 4 visible satellites; In addition, PDOP is usually below 6 if there are no GPS satellite outages (Spilker, 1996). An optimum n-satellite subset from m visible satellites ($m > n$) can be selected based on GDOP to improve the GPS positioning accuracy (Phatak, 2001).

Global Navigation Satellite Systems GPS and GLONASS were constructed and put into use, boosting the reform in the theories of navigation positioning and the application thereof. Subsequently, China and European Union started the construction of BeiDou and Galileo navigation system; Japan started QZSS, and India started IRNSS (Feng and Li, 2008; Yang, 2010). Availability of additional signals besides GPS provides better satellite geometry and diversity that improves navigational accuracy (Butsch, 1997; Wallner et al., 2005; Bhatia et al., 2006; Wallner et al., 2006; Grelier et al., 2007; Yang et al., 2011). Low values of GDOP and PDOP are preferred, and thus the minimum of GDOP and PDOP should be discussed. Langley (1999) discusses the DOP-values and examined the important role that receiver-satellite geometry plays in determining GPS position accuracy. The bounds for DOP-values have been discussed to identify the condition for which the two types of DOP-values coincide (Teunissen, 1998). Sairo et al. (2003) pointed out that the minimum of GDOP is $\sqrt{10/n}$ when there are four visible satellites. But, the minimum of GDOP cannot reach $\sqrt{10/n}$ when receiver is on the earth's surface, because receiver can only receive the signals when the elevation mask angle of satellite is greater than zero. The new minimum GDOP is derived by Han et al. (2013b) under the condition that all visual satellites are above the horizon, and the minimum of GDOP is given under the condition that elevation mask angle of satellite is $[0, \pi/2]$ when the numbers of visual satellites are 4 and 5. Pei et al. (2010) construct a model for the relationship between DOP and the number of stations, and give the situation when the minimum PDOP occurs. Sun et al. (2011) propose a method to establish ISL (inter-satellite link) topology with the minimal PDOP value. So far, the research on the minimum of PDOP is limited. Nowadays, satellite navigation has stepped into an era of rapid development worldwide. With the increase of visual satellites, the minimum of PDOP becomes worthy of research.

This article solves the minimum of PDOP by using matrix knowledge and trigonometric function. The expressions of minimum of PDOP are proposed and the numerical values are also calculated. The paper is organized as follows: Firstly, the definition of GDOP and PDOP is given in Section 2. In Section 3, the conclusions of the minimum of GDOP proposed by the authors are introduced, and then these conclusions are used to study the minimum of PDOP for two cases — when the receiver is on the earth's surface and when the receiver is on satellite. The new theory of minimum of PDOP is used to verify the existing criteria of ISL establishment in Walker-δ constellation in Section 4. In Section 5, the minimum of PDOP is calculated when the number of satellites is 4–10, and these results are verified by using Monte Carlo method (Hammersley, 1960). Finally, some concluding remarks are given at the end of this paper.

2. Definition of GDOP and PDOP

In the not too distant future, positioning, navigation, and timing users are anticipating significant performance improvements as the Galileo system and BeiDou system are fielded. But a large number of additional timing biases will be introduced, including the biases between satellite navigation systems (Hegarty et al., 2004; Wang et al., 2011; Nadarajah et al., 2013). Fortunately, it can be shown that in most cases these biases can be calibrated, and in that case one can work again with the standard GNSS observation equations (Odijk and Teunissen, 2013).

Let ρ^i denote the pseudorange observation between satellite i and receiver at epoch t_k, and the pseudorange observation equation can be written as:

$$\rho^i = D^i + c(\delta t^i - \delta t) + d^i_{orb} + d^i_{ion} + d^i_{trop} + \varepsilon \tag{1}$$

where D^i is the geometric range between the receiver antenna and ith satellite, c is the speed of light, δt^i and δt are the clock errors of the satellite and the receiver respectively, d^i_{orb} is the range error resulted from satellite orbital errors, d^i_{ion} and d^i_{trop} are the ionospheric and tropospheric corrections respectively, ε is the pseudorange measurement noise.

If the receiver continuously observes the same n ($n \geqslant 4$) satellites, through linearization of observation equations around the a priori receiver position $\vec{r}_{uo} = (x_0\ y_0\ z_0)^T$, observations equation becomes, in matrix form:

$$HX = L + \varepsilon \tag{2}$$

where: $H = \begin{bmatrix} e_{x1} & e_{y1} & e_{z1} & 1 \\ e_{x2} & e_{y2} & e_{z2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e_{xn} & e_{yn} & e_{zn} & 1 \end{bmatrix}$ is the $n \times 4$ design matrix, $e_i = (e_{xi}, e_{yi}, e_{zi})$ is the direction cosine of receiver pointing to the ith satellite, and $|e_i| = 1$, that is to say, e_i ($i = 1, 2, \ldots, n$) are all on unit sphere; The n columns of H are linearly independent since they are signals received from individual satellites independently; $X = (\delta x\ \delta y\ \delta z\ b)^T$ is the unknown parameters vector, $(\delta x\ \delta y\ \delta z)^T$ is the vector of position corrections and b is the equivalent distance parameter of the clock error of receiver; L is an $n \times 1$ vector of observations and the weight matrix is unit weight; ε is an $n \times 1$ vector of observation noises.

The least squares estimator of unknown parameters vector X in model (2) can be expressed as

$$\hat{X} = (H^T H)^{-1} H^T L \tag{3}$$