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Effects of perturbations in Coriolis and centrifugal forces on the locations and stability of libration points in Robe's circular restricted three-body problem under oblate-triaxial primaries

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Abstract

This paper investigates the effect of small perturbations in Coriolis and centrifugal forces on the axial libration points and their stability in Robe's circular restricted three-body problem when the hydrostatic equilibrium figure of the first primary is an oblate spheroid; the shape of the second primary is a triaxial rigid body. It is observed that the locations of the axial libration points are only influenced by a small change in the centrifugal force due to the force function Ω being dependent of it. The magnitude of the centrifugal force shift the first libration point $(p_1, 0, 0)$ towards the positive side of the *x-axis*, while second axial libration $(x_{11} + p_2, 0, 0)$ is shifted away from the origin towards the negative part of the *x-axis* as a result of the increase in magnitude of the triaxiality and centrifugal force. The range of stability is established and is found that the first axial libration point is stable for $p_1 < 0$, conditionally stable for $0 < p_1 < \frac{\epsilon}{2}$ and unstable for $0 < \frac{\epsilon}{2} < p_1$, while the second axial libration point is stable whenever $x_{11} < 0$ and if $x_{11} > 0$, then it is conditionally stable.

Keywords: Robe's problem; Perturbations; Oblateness; Triaxiality; Libration points; Linear stability

1. Introduction

The Robe's (1977) restricted three-body problem (R3BP), referred to as Robe's problem, is a peculiar case of R3BP, where the infinitesimal mass is embedded in the first primary which is filled with homogenous incompressible fluid of known density, the second primary is a point mass located outside an orbiting shell. The infinitesimal mass is under the gravitational attraction of the primaries as well as buoyancy force due to the fluid.

Shrivastava and Garain (1991) examined the effect of small perturbations in the Coriolis and centrifugal forces on the position of the equilibrium point in the Robe's circular restricted three-body problem when the densities ρ_1 of the fluid is equal to that the infinitesimal mass ρ_3. After a decade, Hallan and Rana (2001) investigated the work of Shrivastava and Garain (1991) using a different method. Shu and Lu (2005) studied the position and linear stability of the main libration points in Robe's restricted three-body problem under perturbed Coriolis and centrifugal forces. Their work was aimed at improving the work of Shrivastava and Garain (1991) as they argued that the assumption that the densities parameters are equal (i.e., $\rho_1 = \rho_3$) is hard to be fulfilled in practice. For this reason, their study regarded the density parameter as $K \geqslant 0$ (i.e., $\rho_1 \leqslant \rho_3$), they found a unique libration point $(x_0, 0, 0) = \left(-\mu + \frac{\mu\varepsilon'}{1+2\mu-K}, 0, 0\right)$ when $\beta - K \geqslant 0$ and

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estimated the perturbation magnitude of their locations and linear stability.

Hallan and Mangang (2007) examined Robe's problem by considering the buoyancy force, the hydrostatic equilibrium figure of the first primary as an oblate spheroid and considered the full potential due to the second primary. They found conditions for the existence of an infinite number of libration points and their stability. Singh and Sandah (2012) and Singh and Mohammed (2012) extended they work of Hallan and Mangang (2007) by considering the second primary as an oblate spheroid and as a triaxial rigid body, respectively. They derived the pertinent equations of motion and discussed the existence of equilibrium points and their stability. Singh and Omale (2014) extended the work of Singh and Sandah (2012) by considering the shape of the second primary as an oblate spheroid with oblateness coefficients up to the second zonal harmonic. Singh and Cyril-Okeme (2014) examines the existence and linear stability of equilibrium points in the perturbed Robe's circular restricted three-body problem under the assumption that the hydrostatic equilibrium figure of the first primary is an oblate spheroid. Their problem is perturbed in the sense that small perturbations are given to the Coriolis and centrifugal forces are being considered. And they discussed the special case where the density of the fluid and that of the infinitesimal mass are equal ($D = 0$).

The current study will examine the combined effect of perturbations in the Coriolis and centrifugal forces, the full buoyancy force, oblateness of the fluid of the first primary in the shape of an oblate spheroid, and the triaxiality of the second primary on the locations and linear stability of libration points in the Robe's circular restricted three-body problem. We propose to investigate the effect of the small perturbations in Coriolis and centrifugal forces on the axial libration points and their stability.

This Robe's model under consideration will have many applications in various astronomical problems as it may provide insight into the problem of small oscillations of the Earth's core in the gravitational field of Earth–Moon system (since Earth is oblate spheroid and Moon is a triaxial rigid body), the stability of the Earth center as an equilibrium point of the Robe's problem and the motion of the artificial satellites in the Earth–Moon vicinity.

The paper is arranged in five sections; the introduction is in this section. The next section shows the equations of motion. The location of axial libration points is given in Section 3, while their stability is discussed in Section 4 and the conclusion is drawn in Section 5.

2. Equations of motion

Suppose the first primary m_1 be a fluid of density p_1 in the shape of an oblate spheroid and the second primary m_2 which describes a circular orbit around m_1 be a triaxial rigid body as in Singh and Mohammed (2012). The infinitesimal mass m_3 of density p_3 moves inside the first primary. We adopt a synodic coordinate system $\mathbf{0x_1x_2x_3}$ with origin at the center of mass m_1, $\mathbf{0x_1}$ points towards m_2, and $\mathbf{0x_1x_2}$ being the orbital plane of coinciding with the equatorial plane of m_1. Then, the equations of motion of the infinitesimal body of density p_3 in this coordinate system, as in Singh and Mohammed (2012) take the form;

$$\begin{aligned} &\ddot{x}_1 - 2n\alpha\dot{x}_2 = \Omega_{x_1},\\ &\ddot{x}_2 + 2n\alpha\dot{x}_1 = \Omega_{x_2},\\ &\ddot{x}_3 = \Omega_{x_3}, \end{aligned} \tag{2.1}$$

where

$$\Omega = V + \frac{1}{2}n^2\beta[(x_1 - \mu R)^2 + x_2^2],$$

$$V = B + B' - \frac{\rho_1}{\rho_3}\left[B + B' + \frac{1}{2}n^2\beta\left\{(x_1 - \mu R)^2 + x_2^2\right\}\right],$$

$$B = \pi G\rho_1\left[I - A_1(x_1^2 - x_2^2) - A_2x_3^2\right],$$

$$\begin{aligned} B' = {} & \frac{Gm_2}{\left[(R - x_1)^2 + x_2^2 + x_3^2\right]^{1/2}} + \frac{Gm_2(2\sigma_1 - \sigma_2)}{2\left[(R - x_1)^2 + x_2^2 + x_3^2\right]^{3/2}} \\ & - \frac{3Gm_2(\sigma_1 - \sigma_2)x_2^2}{2\left[(R - x_1)^2 + x_2^2 + x_3^2\right]^{5/2}} \\ & - \frac{3Gm_2\sigma_1x_3^2}{2\left[(R - x_1)^2 + x_2^2 + x_3^2\right]^{5/2}}, \end{aligned}$$

$$I = 2a_1^2A_1 + a_2^2A_2,\ \ A_1 = a_1^2a_2\int_0^\infty \frac{du}{(\Delta a_1^2 + u)},\ \ A_2 = a_1^2a_2\int_0^\infty \frac{du}{(\Delta a_2^2 + u)},$$

$$\Delta^2 = (a_1^2 + u)^2(a_2^2 + u),\ \ n^2 = \frac{G(m_1 + m_2)}{R^2}\left(1 + \frac{3}{2}A + \frac{3}{2}(2\sigma_1 - \sigma_2)\right),$$

$$A = \frac{a^2 - c^2}{5R^2},\ \ \sigma_1 = \frac{a^2 - b^2}{5R^2},\ \ \sigma_2 = \frac{b^2 - c^2}{5R^2},\ \ A \ll 1,\ \ \sigma_i \ll 1\ (i = 1,2),$$

$$\alpha = 1 + \varepsilon,\ \ \beta = 1 + \varepsilon',\ \ \varepsilon \ll 1,\ \ \varepsilon' \ll 1,\ \ 0 < \mu = \frac{m_2}{m_1 + m_2} < 1.$$

Here, V stands for the potential that explains the combined forces upon the infinitesimal mass, B and B' denotes the potentials due to the fluid mass and the second triaxial primary respectively. R is for the distance between the primaries, while G stands for the gravitational constant a_1, a_2, and a_3 are the semiaxes of the ellipsoidal figure and parallel to the coordinate axes; A is the oblateness coefficient of the first primary m_1 with semiaxes, while σ_1, σ_2 characterize triaxiality of the second primary m_2 with semiaxes a, b, c. And n is the mean motion. I represent the polar moment of inertia of the oblate primary with index symbol A_1 and A_2. α and β are the small perturbations in Coriolis and centrifugal forces respectively. The last term in V is as a result of the buoyancy force per unit mass, as in Plastino and Plastino (1995), can be expressed as

$$E = -\frac{\rho_1}{\rho_3}\nabla\left[B + B' + \frac{1}{2}\beta n^2\left\{\left(x_1 - \frac{m_2R}{m_1 + m_2}\right)^2 + x_2^2\right\}\right].$$