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A high order method for orbital conjunctions analysis: Monte Carlo collision probability computation

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Abstract

Three methods for the computation of the probability of collision between two space objects are presented. These methods are based on the high order Taylor expansion of the time of closest approach (TCA) and distance of closest approach (DCA) of the two orbiting objects with respect to their initial conditions. The identification of close approaches is first addressed using the nominal objects states. When a close approach is identified, the dependence of the TCA and DCA on the uncertainties in the initial states is efficiently computed with differential algebra (DA) techniques. In the first method the collision probability is estimated via fast DA-based Monte Carlo simulation, in which, for each pair of virtual objects, the DCA is obtained via the fast evaluation of its Taylor expansion. The second and the third methods are the DA version of Line Sampling and Subset Simulation algorithms, respectively. These are introduced to further improve the efficiency and accuracy of Monte Carlo collision probability computation, in particular for cases of very low collision probabilities. The performances of the methods are assessed on orbital conjunctions occurring in different orbital regimes and dynamical models. The probabilities obtained and the associated computational times are compared against standard (i.e. not DA-based) version of the algorithms and analytical methods. The dependence of the collision probability on the initial orbital state covariance is investigated as well.
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1. Introduction

The risk of in-orbit collisions between operative satellites and space debris is a crucial issue in satellite operation. When a close approach is identified, it is necessary to define an indicator that can tell how risky the predicted conjunction is. It is common practice for space agencies and satellite operators to consider, together with conjunction geometry and miss-distance, the collision probability for this purpose (Klinkrad et al., 2005; Righetti et al., 2011).

The collision probability is computed by means of a multi-variate integral. The uncertainties in position and velocity coming from orbit determination can be translated into a probability density function (p.d.f.). The probability density function is then integrated over the volume swept out by the combined hard-body area of the satellite and colliding object, normal to the velocity vector, to retrieve the collision probability.

Different methods exist for the computation of this multi-dimensional integral. Most of these approaches (Akella and Alfriend, 2000; Bèrend, 1999; Patera, 2001;

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Klinkrad, 2006) have the following assumptions in common:

- Position uncertainties of the two objects are not correlated;
- Objects move along straight lines at constant velocity during the conjunction;
- The uncertainty in the velocities is neglected;
- Position uncertainty during the whole encounter is constant and equal to the value during the conjunction;
- The uncertainties in the positions of the two objects are represented by three-dimensional Gaussian distributions.

These assumptions produce accurate results when the relative motion between the satellite and the object is rectilinear and the conjunction occurs close to the initial epoch so that the p.d.f. of the relative position of the two objects remains Gaussian. The probability density function in the proximity of the close approach, under the assumption that position error is Gaussian, is expressed as

$$p(\Delta\boldsymbol{r}) = \frac{1}{\sqrt{(2\pi)^3 \det \boldsymbol{C}}} e^{-\frac{1}{2}\Delta\boldsymbol{r}^T \boldsymbol{C}^{-1}\Delta\boldsymbol{r}}, \tag{1}$$

where $\Delta\boldsymbol{r}$ is the objects relative position vector. Integrating over the volume V swept out by the hard-body sphere with volume V_c, that is the combined volume of the colliding objects, yields the collision probability

$$P_c = \frac{1}{\sqrt{(2\pi)^3 \det \boldsymbol{C}}} \int\int\int_V e^{-\frac{1}{2}\Delta\boldsymbol{r}^T \boldsymbol{C}^{-1}\Delta\boldsymbol{r}}\, dV. \tag{2}$$

Because of the assumption of rectilinear motion of both conjuncting objects, the volume V is a cylinder extending along the relative velocity direction. By integrating the p.d.f. along the cylinder axis from $-\infty$ to $+\infty$, the marginal two-dimensional p.d.f is obtained and the volume integral is reduced to a two-dimensional integral on the collision cross sectional area (Chan, 2008). Supposing that the combined covariance $\boldsymbol{C}$ is centered on the primary object and that the combined hard-body is positioned on the secondary object, the two-dimensional integral of the marginal p.d.f. on the collision cross-sectional area in the (x, y) encounter plane can be written as (Akella and Alfriend, 2000; Klinkrad, 2006; Bèrend, 1999):

$$P_c = \frac{1}{2\pi\sqrt{\det \boldsymbol{C}}} \int_{-R_c}^{R_c} \int_{-\sqrt{R_c^2-x^2}}^{\sqrt{R_c^2-x^2}} e^{-A}\, dy\, dx, \tag{3}$$

where

$$A = \frac{1}{2}\Delta\boldsymbol{r}^T \boldsymbol{C}^{-1}\Delta\boldsymbol{r}, \tag{4}$$

where R_c is the combined radius of the two spherical objects and C now denotes the covariance in the marginal two-dimensional pdf. The analytical methods available in the literature differ in the way the two-dimensional integral is approximated. Chan transforms the two-dimensional p.d.f. into a one-dimensional Rician p.d.f. and uses equivalent areas to develop an analytical approximation of the double integral (Chan, 1997). A series expression to approximate Eq. (3) is derived by Alfano, using a combination of error functions and exponential terms (Alfano, 2006a). In addition, Patera performs an exact reduction of the two-dimensional integral of Eq. (3) to a one-dimensional contour integral over a general-shaped body (Patera, 2001). The method was then extended to use numerical quadrature for a simple one-dimensional integral (Patera, 2005).

Methods that account for non-linearities, which are typical of GEO conjunctions, were also developed (Chan, 2004; Patera, 2003; Patera, 2006). An approach that uses a set of small consecutive linear segments to compute collision probability for non-linear conjunctions is presented in (Alfano (2006b); McKinley, 2006).

The conflict probability, used for air-traffic control by the aviation community (Paielli and Erzberger, 1997), was proposed as an alternative to collision probability as a metric to quantify the collision risk even for space objects (Patera, 2007). The conflict probability is computed similarly to collision probability, using a conflict volume instead of the combined hard-body region. It corresponds to the probability that a single conflict volume, centered on one space object, will be penetrated by the other space object. The conflict volume is large compared to space vehicle size and, as a result, conflict probability is higher than collision probability. In addition, no information on hard-body size, which is usually not available for space debris, is required. The conflict probability was extended to the case of ellipsoidal conflict volumes and tested against other metrics for the identification of risky conjunctions, showing good performances for the analyzed test cases (Patera, 2007).

Besides the analytical methods, the collision probability integral can be computed by means of Monte Carlo (MC) simulations (de Vries and Phillion, 2010; Sabol et al., 2011). Despite being a general and flexible way to compute collision probability, the MC approach has the main drawback of requiring intensive computation, as each virtual satellite/debris trajectory has to be propagated. For this reason Monte Carlo methods are not suitable for daily collision probability computation, since results can be obtained in a timely manner only with simple dynamics, such as two-body propagators or SGP4/SDP4.

In recent times, techniques such as importance sampling (Dolado et al., 2011) or adaptive splitting (Pastel, 2011) have been introduced to cope with the high computational effort. Moreover, a method that couples Monte Carlo with orbital dynamics approximation, obtained by means of polynomial chaos expansion, was introduced to compute satellite collision probability with reduced computational effort (Jones and Doostan, 2013). Monte Carlo methods were also used to study the impact of non-Gaussian probability density functions on collision probability computation (Ghrist and Plakalovic, 2012).

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