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Pulsational mode fluctuations and their basic conservation laws

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Abstract

We propose a theoretical hydrodynamic model for investigating the basic features of nonlinear pulsational mode stability in a partially charged dust molecular cloud within the framework of the Jeans homogenization assumption. The inhomogeneous cloud is modeled as a quasi-neutral multifluid consisting of the warm electrons, warm ions, and identical inertial cold dust grains with partial ionization in a neutral gaseous background. The grain-charge is assumed not to vary in the fluctuation evolution time scale. The active inertial roles of the thermal species are included. We apply a standard multiple scaling technique centered on the gravito-electrostatic equilibrium to understand the fluctuations on the astrophysical scales of space and time. This is found that electrostatic and self-gravitational eigenmodes co-exist as diverse solitary spectral patterns governed by a pair of Korteweg–de Vries (KdV) equations. In addition, all the relevant classical conserved quantities associated with the KdV system under translational invariance are methodologically derived and numerically analyzed. A full numerical shape-analysis of the fluctuations, scale lengths and perturbed densities with multi-parameter variation of judicious plasma conditions is carried out. A correlation of the perturbed densities and gravito-electrostatic spectral patterns is also graphically indicated. It is demonstrated that the solitary mass, momentum and energy densities also evolve like solitary spectral patterns which remain conserved throughout the spatiotemporal scales of the fluctuation dynamics. Astrophysical and space environments significant to our results are briefly highlighted.

Keywords: Star formation; KdV system; Soliton; Conservation laws; Gravito-electrostatic coupling

1. Introduction

This is a well-established fact that the pulsational mode of gravitational collapse of a self-gravitating dust molecular cloud (DMC) is responsible for tremendous amount of energy through star formation processes (Dwivedi et al., 1999; Pandey et al., 2002). It arises due to gravito-electrostatic coupling in presence of partially ionized massive dust grains on the astrophysical scales of space and time. The source of free energy for this gravito-electrostatic instability lies in the associated self-gravity of the dispersed phase of the dust grains of solid matter over the gaseous phase of the background plasma (Pandey et al., 1994, 2002; Dwivedi et al., 1999). Almost all stellar structures and astrophysical objects are the residual end-products of the fluctuations initially started growing due to such a gravito-electrostatic instability (Pandey et al., 1994, 2002; Dwivedi et al., 1999; Klessen et al., 2011; Karmakar and Borah, 2013). In order for the cloud to collapse successfully into a propostar, it is necessary that its gravitational energy exceeds the sum of thermal, rotational and magnetic energies.

The stability of the pulsational mode arising due to gravito-electrostatic coupling in astrophysical situations has extensively been studied by many authors by applying analytical, graphical and numerical techniques (Pandey et al., 1994, 2002; Dwivedi et al., 1999; Karmakar and Borah, 2013 and references therein). This behaves like a time-synchronized resonant linear superposition of a

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purely hybrid oscillator-like mode and a purely growing mode over a levitational equilibrium around the critical Jeans scale length (Dwivedi et al., 1999). The dust-charge fluctuation may decide the degree of pulsational or self-gravitational dominancy of the two modes (Pandey et al., 2002). In contrast, in the nonlinear regime, the gravito-electrostatic fluctuations have been reported to evolve as solitary patterns dictated by the nonlinear equations classified into the Korteweg–de Vries (KdV) family (Asano, 1974; Adams et al., 1994; Karmakar and Borah, 2013). In addition, a strong coupling between the neutral fluid and magnetoplasmas giving rise to novel Jeans-type instabilities in molecular clouds has also been studied by a number of authors (Mamun et al., 1999; Shukla and Mamun, 2000; Mamun and Shukla, 2001). The effects of the inhomogeneities, self-gravity, drag forces, ionization and recombination have been examined to develop dispersive properties of wide-range electromagnetic modes, which could be responsible for the cloud fragmentation into substructures of stellar form (Mamun and Shukla, 2001). But, nobody has so far addressed the fluctuation eigenmodes in presence of the weak but finite inertial effects of the plasma thermal species. Even in presence of the massive grains, the lowest-order inertia of the thermal species has been found to possess a unique quality of destabilizing the normal plasma mode in uniform flow region (Karmakar, 2007). Besides, the conservative nature of the gravito-electrostatic nonlinear fluctuations in diverse astrophysical situations is yet to be well understood and elaborately analyzed.

In this paper the nonlinear stability of the pulsational dynamics in an infinite self-gravitating inhomogeneous cloud (Cartesian 1-D geometry) with all the possible dissipative agencies is studied in hydrodynamic equilibrium. This includes the lowest-order inertial correction (Dwivedi and Prakash, 2001; Deka et al., 2004; Deka and Dwivedi, 2010) of the thermal species (hereafter, termed as "thermal inertia"). We develop a systematic methodology to demonstrate a full existence of new nonlinear eigenmode features of gravito-electrostatic significance with all the possible realistic collisional effects (Verheest, 1996; Crutcher, 2012) taken into account. Applying a standard multiscale analysis, it is shown that the electrostatic and self-gravitational eigenmodes evolve as diverse solitary spectral patterns governed by a pair of gravito-electrostatically coupled Korteweg–de Vries (KdV) type equations. A full numerical shape-analysis of the fluctuations, scale lengths and densities with multi-parameter variation of judicious plasma conditions is also carried out to study the internal microphysics. Moreover, the KdV system is well-known as a conservative integrable model possessing an infinite string of conservation laws (Miura et al., 1968; Newell, 1985; Benguria and Depassier, 1989). So, motivated by the conservative features of the fluctuations, we analytically derive different basic conservative forms of the pulsational KdV dynamics. The evolutionary patterns of all the associated relevant conserved quantities are numerically analyzed. Interestingly, we find a unique property that the dynamical evolution of the mass density, momentum density and energy density of the lowest-order gravito-electrostatic potential fluctuations retain the shape of soliton-like patterns in all cases. The layout of the paper is organized as follows. In addition to "introduction" part as described in Section 1 above, Section 2 contains the physical model and the basic governing equations of the cloud structure. Section 3 portrays the derivation of the basic conservation laws associated with the fluctuations, which is again subdivided into two Sections 3.1 and 3.2, depicting the electrostatic and self-gravitational KdV dynamics, respectively. Last of all, Section 4 presents the summary and main conclusions of space and astrophysical significance together with some highlighted future directions on further applicability.

2. Physical model and mathematical formulation

A simplified idealistic field-free one-dimensional (1-D Cartesian) cloud is considered under hydrodynamic equilibrium configuration with dust flow convection and collisional dynamics included. The solid matter of the identical spherical dust grains is embedded in the gaseous phase of quasi-neutral plasma on the Jeans scale, and thereby gets partially ionized. A bulk uniform flow is assumed to preexist in hydrodynamic equilibrium configuration. Global electrical quasi-neutrality is supposed to exist over the gravito-electrostatic enclosure containing the diverse plasma particles. On the Jeans scales of space and time, heavier dust grains (inertial) are assumed to behave as a hydrodynamic fluid, whereas inertia-corrected electrons and ions are supposed to behave as thermal species (very less inertial). Thus, the gravitational mass of the cloud as a whole is collectively contributed essentially by the grains only.

The gravitational decoupling of the background neutral particles under consideration is justifiable for higher inertial mass of the grains with higher neutral population density so that the Jeans mode frequency becomes reasonably large. The physical basis of our model is the Jeans assumption of self-gravitating uniform medium adopted for fiducially analytical simplification by neglecting the zero-order field. Consequently, the gravito-electrostatic equilibrium is justifiably treated initially as a "homogeneous" one. Since, the zero-order self-gravitational field is neglected, where the equilibrium is treated initially as "homogeneous", it thereby validates local analysis, very close to the realistic scenario (Cadez, 1990; Vranjes, 1994; Dwivedi et al., 1999; Pandey et al., 2002; Verheest et al., 2002; Falco et al., 2013). For further ease, we neglect rotation, viscosity, magnetic field, diffusion, charge fluctuation, grain-size distribution, variation in temperature, etc. (Verheest, 1996; Crutcher, 2012).

We are interested to investigate the nonlinear electro-gravitational stability of the pulsational mode dynamics in presence of the inertia-corrected thermal electrons and ions, and their basic conservation laws. So, the lowest-order