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Eccentric dipole approximation of the geomagnetic field: Application to cosmic ray computations

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Abstract

A comparison of the full IGRF model of the geomagnetic field with two simplified models, the truncated IGRF and the eccentric dipole model, is performed. The simplified models were found to provide a reasonable approximation for the large scale geomagnetic field distribution. In the application of the simplified geomagnetic models to the shielding of cosmic rays in the magnetosphere as quantified via the geomagnetic cut-off rigidity, the eccentric dipole and the truncated IGRF provide a good large scale view. The use of the simplified model does not introduce any additional systematic errors at the global scale but may be a source of moderate uncertainty at the regional scale in the tropical Atlantic region. This study quantitatively validates the use of such simplified geomagnetic models when describing the shielding of cosmic rays in the magnetosphere.

Keywords: Cosmic rays; Geomagnetic field; Paleomagnetism

1. Introduction

The geomagnetic field effectively shields the Earth from incoming cosmic rays – highly energetic nuclei of extra-terrestrial origin. Since cosmic rays are charged particles, their trajectories are bent in the geomagnetic field, leading to shielding, so that energetic particles need to possess minimal energy to be able to penetrate through the field towards Earth. The shielding depends on the direction of the geomagnetic field so that it is stronger in the equatorial region, where the magnetic field lines are tangential to the Earth's surface, and absent in the polar regions where the magnetic lines are vertical. Thus, the shielding is unevenly distributed over the globe.

In order to study the cosmic-ray induced effects in the Earth's atmosphere, such as cosmic ray induced ionization (e.g., Bazilevskaya et al., 2008) or production of cosmogenic radionuclides (e.g. Beer, 2000), one has to account properly for the geomagnetic shielding. This can be done straightforwardly for the recent epoch, when the geomagnetic field is well measured and known. This is normally done via the concept of the geomagnetic cutoff rigidity (Cooke et al., 1991), viz. the minimal rigidity (momentum over charge) a charged particle must possess to be able to reach the ground in the absence of the atmosphere. For the recent times, last century or so, covered by extensive geomagnetic measurements, the cutoff rigidity can be calculated (Smart et al., 2000; Shea and Smart, 2001) using the IGRF (International Geomagnetic Reference Field – see Section 2.1) with the full information on multipole components of the geomagnetic field. However, for more distant past, when direct geomagnetic measurements were not performed, one has rely upon paleo- or archeo-magnetic reconstructions (Genevey et al., 2008; Donadini et al., 2010), which provide less information on the higher harmonics of the field. In such a condition a simplified approach is used to assess the geomagnetic shielding of cosmic rays. It is typical to represent the geomagnetic field

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only by its dipole components, which are known much better than the regional field structure in the past (Korte and Constable, 2005; Korte et al., 2011). Reconstructions of the global geomagnetic field at the millennial scale can resolve only the large-scale (Korte and Constable, 2008; Genevey et al., 2008). When spherical harmonic models are available, the contributions from dipole and quadrupole, or of an eccentric tilted dipole can be considered as described in Section 2.2. Although this is considered a reasonable approximation (Bartels, 1936; Elsasser et al., 1956; Fraser-Smith, 1987; Lowes, 1994; Olson and Deguen, 2012), a question of quantitative assessment of the possible uncertainties related to the use of such a simplified approach is still open.

In this paper we compare the geomagnetic cutoff rigidity calculated using different models of the geomagnetic field and assess their uncertainties and validity.

2. Geomagnetic models

The geomagnetic field has a complicated structure, which also depicts slow temporal variability. There are different ways to describe it mathematically. Here we review two ways: the IGRF and the eccentric dipole model.

2.1. IGRF

The International Geomagnetic Reference Field (IGRF) is a reliable standard model which represents the large scale internal part of the geomagnetic field on and above Earth's surface (Finlay et al., 2010). The IGRF model parameters are added periodically for a next epoch of five years so that the parameters are interpolated/extrapolated between the five-year epoches. The preceding IGRF parameters can be updated and become DGRF parameters, and parameters for the extrapolation over the next five years are published by the IAGA Working Group V-MOD (`http://www.ngdc.noaa.gov/IAGA/vmod/`). The model is derived from observations collected by satellites, at observatories at land and during magnetic surveys. The parameters for the IGRF model are available since 1900 AD, and the current IGRF model is eleventh generation dated on 2010 and is valid until 2015, when the next generation is to be released.

The IGRF model uses the multipole representation of the geomagnetic field based on an assumption that the density of current between the surface and ionosphere is negligible near the surface, so that the field can be taken to be curl-free. This allows the field **B** to be presented as the gradient of a scalar potential V (Jacobs, 1991)

$$\mathbf{B} = -\nabla V \tag{1}$$

The scalar potential V is represented through a finite series of numerical Gauss (spherical harmonic) coefficients g_n^m and h_n^m of degree n and order m, which represent multipole (dipole, quadrupole, etc.) components, centered at the Earth center and aligned with the geographical axis.

$$V(r,\theta,\phi,t) = a\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}\left(g_n^m(t)\cos(m\phi) + h_n^m(t)\sin(m\phi)\right) \cdot P_n^m(\cos(\theta)) \tag{2}$$

where r, θ, ϕ, t are the geocentric distance, geographic colatitude and east longitude of the given location, and time, respectively. P_n^m are the associated Legendre polynomials.

The full IGRF model uses about 200 Gauss coefficients, corresponding to multipoles up to degree and order 13, before 2000 ten multipoles were used. We will henceforth refer to the results based on this full model as IGRF. However, in the past paleomagnetic reconstructions are able to provide less detailed information, the most reliably resolving dipole and quadrupole components, corresponding to an approximation based on the centered aligned dipole and quadrupole. We will refer to the results based on this truncated IGRF model as t-IGRF.

2.2. Eccentric dipole model

The Eccentric dipole approximation (Fraser-Smith, 1987; Olson and Deguen, 2012) also uses the first eight Gauss coefficients of the geomagnetic field representation but arranges them differently. It considers only a magnetic dipole which is however displaced from the Earth center and tilted with respect to the geographical axis. The magnetic dipole moment is defined using the first three Gauss coefficients g_1^0, g_1^1 and h_1^1, while five higher order coefficients define the displacement and the tilt of the dipole (see formalism in the Appendix of Usoskin et al., 2010). We will refer to the results based on this model as ED.

3. Comparison of geomagnetic models

In this section we compare the three different geomagnetic models, viz. IGRF, t-IGRF and ED, at different distances from the Earth's surface.

3.1. Total field

The magnetic field representations by the three considered models for the epoch of 2010 are shown in Figs. 1 and 2, for the Earth's surface and 10 Earth radii away, respectively. Panels A through C stand for the IGRF, t-IGRF and ED models, respectively.

While all the models correctly reproduce the main pattern of the surface large scale field, including the South Atlantic Anomaly and the sigmoid shape of the geomagnetic equator, there are some regional features that truncated models cannot catch. However, the discrepancy quickly fades away as the distance from the surface increases. All the three plots are nearly identical at 10 Earth radii (Fig. 2). Already at a few radii above the surface, hardly any essential difference exists between the models. This is quantified in Table 1, which shows the difference between the IGRF and ED models as a function of the