Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 55 (2015) 545–553

ADVANCES IN SPACE RESEARCH (a COSPAR publication)

www.elsevier.com/locate/asr

Study of low-latitude ionosphere over Indian region using simultaneous algebraic reconstruction technique

S. Nandi [a,*], B. Bandyopadhyay [b,1]

[a] Department of Electronics & Communication Engineering, Jalpaiguri Govt. Engineering College, Jalpaiguri 735102, West Bengal, India
[b] Institute of Radio Physics & Electronics, University of Calcutta, 92 APC Road, Kolkata 700009, India

Received 2 July 2014; received in revised form 14 October 2014; accepted 16 October 2014
Available online 23 October 2014

Abstract

Recognizing the advantages of simultaneous algebraic reconstruction technique (SART), it is applied to investigate the electron density distribution of low-latitude ionosphere. The total electron content data measured at three receivers in West Bengal, India, are used in this analysis. The efficiency of SART in the reconstruction of large scale ionospheric structure is addressed through reconstruction of ionization anomaly. Comparison with the model ionospheric data ensures the accuracy of the SART method for the reconstruction at low-latitude region. Also, a case study with different reconstruction geometry reveals the ability of fast convergence of simultaneous algebraic reconstruction algorithm irrespective of the geometry.

Keywords: Ionosphere; Total electron content; Tomography; SART

1. Introduction

Ionosphere is an important part of the Earth's upper atmosphere. A significant number of neutral atoms are ionized in this region resulting free electrons and ions forming the plasma. Electromagnetic wave when passes through this ionization region suffers reflection, refraction, time delay and also rotation of the plane of polarization. To analyze these effects, it became necessary to study the structure of the ionosphere. The most important physical parameter required to analyze the ionospheric effects is the electron density. Ionospheric imaging of electron density provides a pictorial representation of the plasma structure. The problem of imaging, i.e., reconstruction of a function from its projection, was originally solved mathematically by Radon (1917) and applied to radio astronomy by Bracewell (1956). Application of imaging to the ionosphere involves measurement of total electron content (TEC) of the ionization region. The TEC measurements can be obtained from the networks of global positioning system (GPS) receivers located around the earth. These data can be directly used to retrieve the information of the structure of ionosphere.

Tomographic inversion technique is an example of imaging performed from diverse measurements. The technique finds its application in the area of biomedical diagnostics, (Hounsfield, 1972). Further applications can be found in the book by Kak and Slaney (2001). Method of computerized ionospheric tomography (CIT) first proposed by Austen et al. (1986, 1988), became an important and useful method to study the structure of the ionosphere. Further, a good overview over the ionospheric applications (Raymund, 1995) and geophysical applications (Ivansson, 1986) attracted the scientists for study the ionospheric propagation characteristics in details using CIT. Till then, a number of studies have been reported (Andreeva et al., 1990; Kunitsyn and Tereshchenko, 1992; Kunitake et al.,

* Corresponding author. Tel.: +91 9830382398; fax: +91 (3561)256146.
E-mail addresses: sandip622@yahoo.co.uk (S. Nandi), bb.rpe@caluniv.ac.in (B. Bandyopadhyay).
[1] Tel.: +91 9830504902; fax: +91 (33)23515828.

http://dx.doi.org/10.1016/j.asr.2014.10.018
0273-1177/

1995; Markkanen et al., 1995; Mitchell et al., 1995; Pryse et al., 1995; Huang et al., 1998) to reconstruct an image of the ionospheric electron content in horizontal and vertical plane over different regions. Some of widely used CIT algorithms are the algebraic reconstruction technique, ART (Austen et al., 1988), the multiplicative algebraic reconstruction technique, MART (Raymund et al., 1990; Kersely et al., 1993), the simultaneous algebraic reconstruction technique, SART (Andersen and Kak, 1984), regularization technique (Lee et al., 2007), neural network method (Ma et al., 2005), Kalman filters (Hernandez-Pajares et al., 1999; Ruffini and Rius, 1998) and singular value decomposition, SVD (Bhuyan at al., 2004). All the above methods have their individual shortcomings. Yeh and Raymund (1991) and Raymund et al. (1994) discussed some of the limitations of ionospheric imaging by tomography.

Among all the above mentioned methods, SART has proven to be the most useful iterative reconstruction technique (Ming and Ge, 2003) and better suited for real time applications. The major advantage of SART is that, even in high resolution tomographic problems, number of unknowns can be solved with less computational load. The number of operations in SART is determined by the number of unknowns (N), whereas in non-iterative approaches, number of operations are determined mainly by the size of the design matrix ($N \times N$). Sometimes the situation arise when number of equations (M) becomes smaller than the number of unknowns (i.e., $M < N$), and the system is under determined. Further, a not uncommon situation is when $M > N$ i.e., the system is over determined. In both the situations, no unique solution for the system of equation exists and the best solution which is closest to the initial guess has been chosen. Tanabe (1971) proved that when $M < N$, the iterative approach converges to a solution such that the error is minimized. Hobiger et al. (2008) mentioned that SART has the advantage that it always iterates to converge to a unique solution irrespective of the above situations. No further study has been reported yet to verify this statement.

In this paper, the electron density distribution of low-latitude ionosphere along the 88.34° E meridian is investigated over the region West Bengal, India through tomographic reconstruction using simultaneous algebraic reconstruction algorithm. We intend to address the efficiency of SART in the reconstruction of large scale ionospheric structure. The algorithm is also tested for its ability to converge towards a unique solution irrespective of the geometry of reconstruction. The entire work has been carried out with real observations from GPS receivers and IRI 2012 model data.

2. Reconstruction Method

2.1. Theory

Reconstruction of a function from its projection was a great mathematical issue solved by Radon (1917) and first applied to radio astronomy by Bracewell (1956). The tomographic imaging procedure has been applied to the ionosphere for reconstruction of the electron density profile subject to the advantage that the measurements can be approximated as line-integrals of the electron density. The number of free electron contained within a column of unit cross sectional area in the direction from receiver (R) to the satellite (S) is the slant TEC along the path and can be expressed as

$$STEC = \int_S^R N\, ds + e_N + c \tag{1}$$

where $STEC$ is the slant TEC measured along the path, N is the electron density, ds is the differential distance along the path from S to R, e_N is error term arises due to multipath noise and observation noise, and c is an arbitrary constant that includes the anonymous differential phase cycle at the start of the measurement. Determination of c requires TEC calibration aided with differential code technique (Mannucci et al., 1999). This technique dictates to include the receiver and satellite inter frequency biases with the measurement. Thus the observation Eq. (1) can be read as

$$STEC_{obs} = STEC + B_S + B_R \tag{2}$$

where B_S and B_R are satellite and receiver biases respectively.

The 2-dimensional ray tomography approximates the ionosphere as a two dimensional cross section of the three dimensional space containing the receiver and satellite, and discretizes the plane in N number of pixels which we can call as inversion region. For M number of rays from satellite to receiver, the tomography problem is transformed to a problem of solving linear system of equations

$$P_{M\times 1} = W_{M\times N} f_{N\times 1} + e_{M\times 1} \tag{3}$$

where $W_{M \times N}$ is the transition matrix for the function, $f_{N \times 1}$, the reconstructed function, $P_{M \times 1}$ is the line integrals called the ray-sum generated from the observed STEC, in Eq. (2). The matrix W has different forms depending on the methods used for reconstruction and also on the approximations used (Kunitsyn et al., 1994, 1995a, 1995b).

In this paper, the matrix W has been generated by approximating the function over the image cells of the inversion region. The factor W_{ij} is equal to the length of the ith ray intercepting the jth image cell. Thus, the first objective is to find out the length of the ray paths in each cell and estimate the TEC for a particular receiver station. Then SART iterative algorithm is used to solve Eq. (3) for minimization of errors between estimation and measurement for a particular receiver station.

2.2. Simultaneous algebraic reconstruction technique

The linear imaging problem described by Eq. (3) has been solved using SART (Andersen and Kak, 1984) and is given by