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A high order method for orbital conjunctions analysis: Sensitivity to initial uncertainties

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Abstract

A high order method to quickly assess the effect that uncertainties produce on orbital conjunctions through a numerical high-fidelity propagator is presented. In particular, the dependency of time and distance of closest approach to initial uncertainties on position and velocity of both objects involved in a conjunction is studied. The approach relies on a numerical integration based on differential algebraic techniques and a high-order algorithm that expands the time and distance of closest approach in Taylor series with respect to relevant uncertainties. The modeled perturbations are atmospheric drag, using NRLMSISE-00 air density model, solar radiation pressure with shadow, third body perturbation using JPL's DE405 ephemeris, and EGM2008 gravity model. The polynomial approximation of the final position is used as an input to compute analytically the expansion of time and distance of closest approach. As a result, the analysis of a close encounter can be performed through fast, multiple evaluations of Taylor polynomials. Test cases with objects ranging from LEO to GEO regimes are considered to assess the performances and the accuracy of the proposed method.

Keywords: Space debris; Conjunction analysis; Differential algebra

1. Introduction

The detection of orbital conjunctions between spacecraft and space debris is of fundamental importance in space situational awareness (SSA) programs. Once a potentially dangerous closest approach is identified, for instance by looking at the minimum distance between the objects, all information required to analyse the conjunction is provided to satellite operators that will compute the collision probability and evaluate the collision risk.

The collision risk depends on the geometry of the encounter and is considerably affected by the uncertainties of the orbital states at the time of closest approach (TCA). These uncertainties, in turn, depend on the uncertainties on initial state and their evolution along the orbit. When the TCA is far away from the reference epoch of the initial state, nonlinearities can play an important role in the computation of collision probability, since the initial covariance ellipsoid stretches and deforms after each revolution.

In the past, the conjunction assessment procedures relied on the catalog of unclassified objects orbiting the Earth, maintained by the United States Strategic Command (USSTRATCOM). The catalog is still available nowadays and contains all up-to-date two-line elements (TLE), that are intended for the use with the SGP4/SDP4 orbital model (Hoots et al., 2004). The Center for Space Standards and Innovation (CSSI) produces daily reports of closest conjunctions for the upcoming week using the program Satellite Orbital Conjunction Reports Assessing Threatening Encounters in Space (SOCRATES) (Kelso and Alfano, 2005). The information is publicly posted at www.celestrak.com/SOCRATES/. The conjunctions are identified using TLEs and SGP4/SDP4 and the tool also

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computes the maximum conjunction probability (Alfano, 2005), which represents the upper bound of collision probability and is obtained assuming the worst-case orientation and size of the covariance matrices.

When using TLEs and the SGP4/SGP4 analytical propagator, it has to be taken into account that the time of closest approach (TCA) can vary up to tens of seconds and the distance of closest approach (DCA) up to a few kilometers if more recent TLEs are considered for conjunction detection. Propagation accuracy of SGP4/SDP4 is indeed object dependent, and after a few days of propagation the errors can easily exceed tens of kilometres. In addition, potentially significant biases exist in TLE data (Kelso, 2007).

Nevertheless, when precise ephemeris of the chasers are not available, TLEs can represent a significant source of information. It is possible to estimate covariance information for TLEs by comparing states derived directly from the TLE data with states resulting from an orbit determination using pseudo-observations derived from TLE data (Alarcon-Rodriguez et al., 2004). Alternatively, it is possible to derive a covariance matrix differencing a set of TLEs propagated up to a common time (Laporte and Sasot, 2008; Vallado and Cefola, 2012).

Besides the publicly available TLE catalog, the Joint Space Operation Center (JSpOC) maintains an High-Accuracy Special Perturbation Catalog (Coffey et al., 1998) and releases Conjunction Summary Messages (CSM) to warn satellite operators of incoming close conjunctions (Aida and Kirschner, 2012). The CSM also provides the chaser covariance matrices at the TCA which can be used, together with satellite ephemeris, for accurate collision risk assessment.

The operational service for the assessment of collision risks of ESA satellites is based on the collision risk assessment software CRASS and the orbit determination software ODIN (Flohrer et al., 2009a). A daily automated screening is performed to identify close approached between covered missions and TLE from USSTRATCOM. When the estimated collision probability for an encounter exceeds a given threshold further data are acquired by the operator and processed by ODIN to improve orbit and covariance information. Methods were developed to estimate uncertainties associated with TLEs (Flohrer et al., 2009b). The process was adapted to take into account the CSM and analysis were performed to verify CSM against conjunction event analysis based on radar measurements (Flohrer et al., 2013).

In this work, a method for the computation of the DCA and TCA for all the objects compatible with the initial orbital uncertainties (referred to as virtual debris in the remainder of the paper) with a single numerical integration is presented. The method is based on the high order Taylor expansion of the flow of the dynamics enabled by differential algebraic (DA) techniques. In particular, a DA-based integrator and a polynomial inversion algorithm are used to express the dependence of TCA and DCA on orbital uncertainties in terms of high order Taylor polynomials (Armellin et al., 2010). As a result, the multiple integrations required by a Monte Carlo based approach for the computation of TCA and DCA for all the virtual debris are substituted by fast polynomial evaluations. The computation of these polynomials and the study of their accuracy is the main focus of the paper. Their use for the fast and accurate computation of collision probability will be addressed in future work.

The paper is organized as follows. Firstly, some notes on DA techniques and the method for the high order expansion of the flow are introduced. A description of the dynamical model used follows. Then, the algorithm developed for the Taylor expansion of the TCA and DCA is presented. The performances of the numerical propagator in terms of computational time and expansion accuracy are first illustrated. Subsequently, the attention is focused on the accuracy analysis of TCA and DCA expansions.

2. Notes on differential algebra

DA techniques, exploited here to obtain kth order Taylor expansions of the flow of a set of ODE's with respect to the initial conditions, were devised to attempt solving analytical problems through an algebraic approach (Berz, 1999b). Historically, the treatment of functions in numerics has been based on the treatment of numbers, and the classical numerical algorithms are based on the mere evaluation of functions at specific points. DA techniques rely on the observation that it is possible to extract more information on a function rather than its mere values. The basic idea is to bring the treatment of functions and the operations on them to a computer environment in a similar manner as the treatment of real numbers. Referring to Fig. 1, consider two real numbers a and b. Their transformation into the floating point representation, $\overline{a}$ and $\overline{b}$ respectively, is performed to operate on them in a computer environment. Then, given any operation $*$ in the set of real numbers, an adjoint operation $\circledast$ is defined in the set of floating point (FP) numbers so that the diagram in Fig. 1 commutes. (The diagram commutes only approximately in practice due to truncation errors.) This means, transforming the real numbers a and b into their FP representation and operating on them in the set of FP numbers returns the same result as carrying out the operation in the set of real numbers and then transforming the achieved result in its FP representation.

In a similar way, let us suppose two k differentiable functions f and g in n variables are given. In the DA framework, the computer operates on them using their kth order Taylor expansions, F and G respectively. Therefore, the transformation of real numbers in their FP representation is now substituted by the extraction of the kth order Taylor expansions of f and g. For each operation in the space of k times differentiable functions, an adjoint operation in the space of Taylor polynomials is defined so that the corresponding diagram commutes; i.e., extracting the Taylor expansions of f and g and operating on them in the space