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Planar fly-by trajectories to Moon in the restricted three-body problem

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Abstract

Planar fly-by trajectories from Earth to Moon are studied in the framework of restricted three-body problem (R3BP). A number of trajectories with initial conditions $(x, y, \dot{x}, \dot{y})$ of the form $(x_0, 0, 0, \dot{y}_0)$ and $(-\mu, y_0, \dot{x}_0, 0)$ close to Earth are propagated for 1000 days. The distribution of the initial conditions which will generate feasible fly-bys to Moon in the phase space are plotted in Jacobi constant, C verses initial x or y plots with color codes on time of flight. It is observed that the flyby trajectories with similar flight duration appear in clusters and show some distinctive patterns. For example for the initial condition of first type the fly-by trajectories with flight duration of about 14 to 16 days appear as a curve in the phase space. While for the second type of initial conditions the trajectories with flight duration $\leqslant 5$ days and 5–15 days appear as a band on the phase space. Poincaré surface of section technique is used to identify the existence of periodic and quasi-periodic orbits in other regions of the phase space.

Keywords: Restricted three-body problem; Poincaré surface of section; Planar fly-by trajectories; Jacobi constant; Flight duration

1. Introduction

A conventional method to obtain an initial lunar trajectory is by using the patched conic technique in the two-body problem. In the initial phase, the gravitational influence of Moon is neglected and in the final phase when the spacecraft is within the sphere of influence of the Moon, the gravitational influence of Earth is neglected. After the initial trajectory is finalized it is improved by numerical integration of the full-force equations of motion which mainly takes into account the asphericity of Earth, solar and lunar gravity, atmospheric drag and solar radiation pressure. The time of flight for such trajectories varies from 3–6 days (Battin, 1999). The initial trajectory estimate will be better if gravitational attraction due to Earth and the Moon are considered during the entire trajectory. This brings the problem to three-body problem (Earth-Moon-spacecraft). The spacecraft is of negligible mass and so its influence on the motion of the primaries, (namely the Earth and the Moon, in this case) can be neglected. This is the restricted three body problem (R3BP). This also gives only an initial estimate because of assumptions like the circular motion of the primaries and neglecting Sun's perturbation, atmospheric drag etc., and so the trajectory has to be improved using the full-force models. The trajectories obtained from the R3BP are the natural routes existing due to the interplay of the gravitation of two massive bodies. Many attempts have been made and are still being made to obtain lunar trajectories in the framework of the R3BP which require lesser fuel compared to Hohmann transfer trajectories and take less flight duration.

Egorov (1965) deals with various aspects of lunar trajectory design and selection of the optimum. Hadjidemetriou (1968) classified the two- and three-dimensional orbits through L1 according to their first minimum distance from either primary. Belbruno (1987) and Belbruno and Miller (1993), introduced the new concept of lunar transfer using ballistic capture at the Moon. There are also concepts of switching between different orbit arcs, to achieve low energy transfer trajectory with less impulse (Koon et al.,

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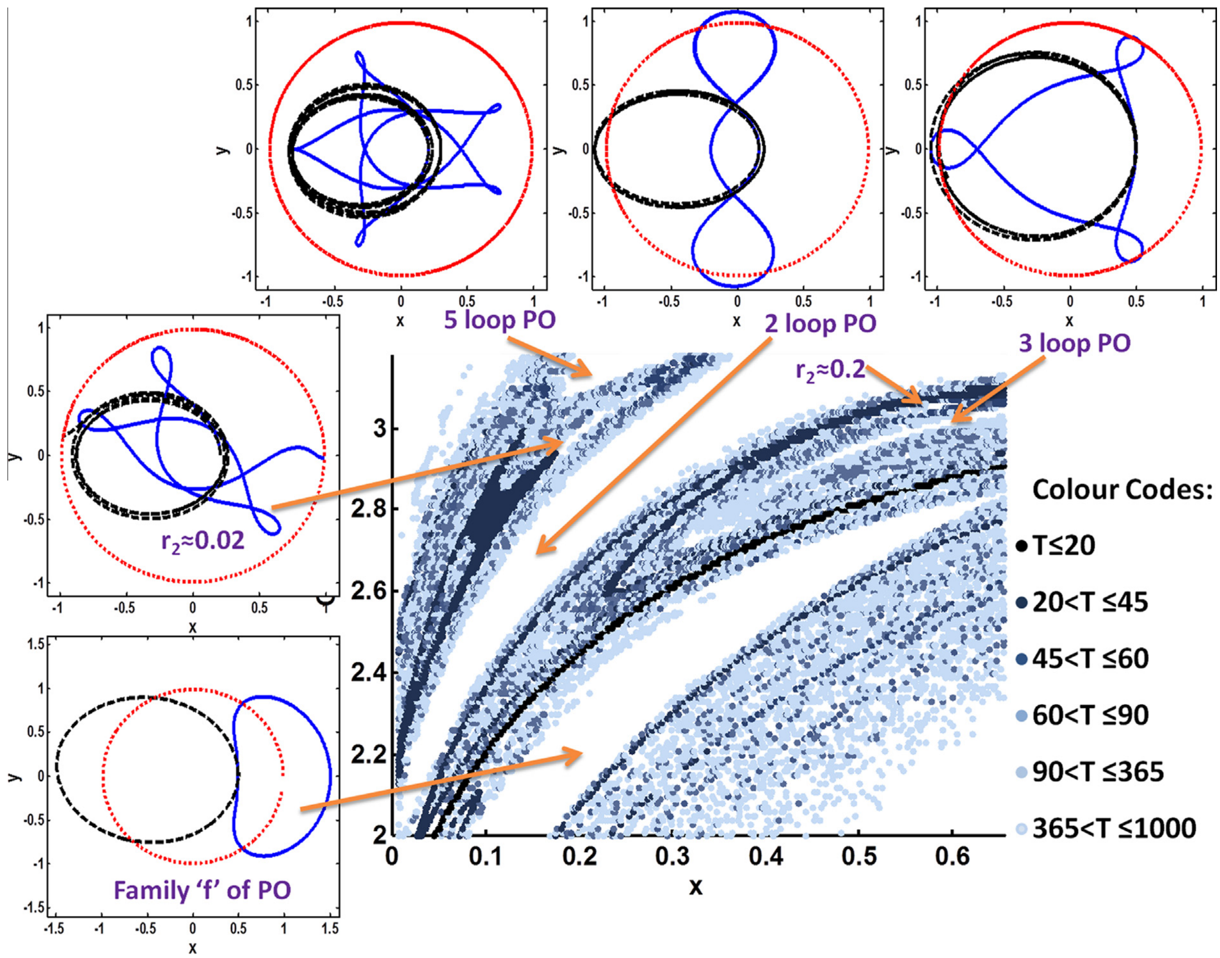

Fig. 1a. Fly-by time in the initial condition space of C versus x. Time in days is represented by 6 different colors. The white regions inside the range of initial conditions correspond to the trajectories that do not reach near to the Moon up to 2000 km from surface of Moon in 1000 days. Some of the representative trajectories are shown in the surrounding figures. In the figures containing the trajectories, solid blue line denotes the trajectory in synodic system, black dashed line denotes in non-rotating co-ordinate system and red dotted line shows the path of Moon. (a) Initial condition space for set (i) of ICs along with the periodic orbits contained in the white regions.

2011). Yagasaki (2004) finds single transfer orbit from Earth to Moon which minimizes impulse requirements but at the cost of time of flight. Briozzo and Leiva (2004) have presented an atlas of more than 70 families of low energy periodic orbits encircling the Earth and the Moon with time of flight less than six months. Melo et al. (2007) have identified stable, escape and capture trajectories through the L1 and L2 by back propagation, i.e., starting near Moon they have back propagated in time till the energy of the particle in two-body Moon-particle system remains negative. They have presented these trajectories in plots of eccentricity versus semi-major axis as a function of time. Similar gravitational capture is studied in the framework of the Sun-Uranus-particle system by Neto and Winter (2001) and Winter and Neto (2001).

To understand the dynamics of transfer trajectories and their distribution in the phase plane, in this paper a sample of initial conditions close to Earth are propagated for 1000 days, an upper bound for practical purposes. The initial conditions are of the form $(x_0, 0, 0, \dot{y}_0)$ and $(-\mu, y_0, \dot{x}_0, 0)$. During propagation if the minimum distance of the spacecraft from the surface of Moon is less than 2000 km, then that trajectory is recorded. Poincaré surface of section is used to identify the existence of periodic and quasi-periodic orbits in these regions. This method is different from the others because here the initial conditions are propagated forward in time for 1000 days irrespective of whether they reach near Moon or not. This enables us to identify the regions containing transfer trajectories to Moon in the phase space with respect to transfer time (within 1000 days) which is not available in previous works. Presently, only a sample of initial conditions has been analyzed in the planar problem. The method can be generalized for more initial conditions and for inclined orbits. Moreover perturbations from Sun can also be incorporated and compared with these results.

2. Equations of motion

The motion of the spacecraft is simulated by numerically integrating the planar circular R3BP. This defines the motion of the spacecraft on the plane of motion of